# The Valuation of Private Companies

## Asset valuation and the dynamics of private markets

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# Contents

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# Executive Summary

Investors face tremendous problems when attempting to value investments in unlisted private companies. Such entities lack reliable valuation histories, preventing the construction of meaningful, accurate, and representative indices in private markets. In addition, currently available benchmarks typically rely on appraised valuations, which have many drawbacks. They are subject to several well known biases, such as smoothing, staleness, and poor representativeness, and do not reflect all private companies or all the information available in private markets.

Due to this lack of robust benchmarks, many investors perform comparable analyses (or comps) using the available transaction data on private companies. However, such data is sparse, noisy, and biased because few private companies transact regularly and each valuation is subject to idiosyncratic (or individual) factors. Therefore, comps based on such data are not very useful to investors.

In this paper, we propose a solution that is neither based on appraised valuations nor subject to these biases in transaction data. The factor model based solution when calibrated with transaction data and novel risk factors proposed in the paper, can transform the sparse, noisy, and biased transaction data into meaningful information that aids asset allocation, benchmarking, and monitoring of private investments.

**First**, by being based on risk factors that are captured in each transaction, the model learns from each transaction, unlike comps where only the price data from the transactions is included.

**Second**, by estimating the factor models recursively (i.e., dynamic linear models), the time-varying factor premiums are captured, thus accommodating more of the dynamics of private market valuation. For example, the price investors pay for a private company of a certain size (i.e., size preferences) may change over time, such as during recessions, thus altering the effect of size on valuation ratios. The proposed factor model explicitly accounts for such variations in preferences and is able to estimate true and unbiased factor prices even when calibrated using biased samples.

**Third**, the lack of standardised and strict regulatory requirements governing private companies across countries leads to data availability concerns that hinder the construction of risk factors. The data that is already available for private companies can be augmented using the PECCS™ or *PrivatE Company Classification Standard* taxonomy; this is a rigorous and objective classification scheme that goes beyond industrial activities and captures several key risk factors, by grouping private companies across several dimensions of risks. PECCS™ incorporates independent pillars of industrial activity, lifecycle phase, revenue model, customer models, and value chain.

For example, using PECCS™, the factor model can capture risk factors associated with startup companies by estimating a higher valuation when start-up companies transact expensively, or vice versa, irrespective of their industrial sector. Likewise, companies with subscription type revenue models may command a premium valuation to peers with different revenue models. Through the PECCS™ classification, such variations are captured in the factor model, thus accommodating a broad set of risk factors whilst dealing with the limited data availability constraints presented by private markets.

This factor model enables investors to perform valuations of private companies while avoiding the pitfalls of relying on appraisal data and the arbitrariness of using raw transaction data. The model provides a flexible framework to value the universe of private companies and facilitates the building of robust benchmarks – the kind investors are used to in publicly traded markets.

Moreover, the factor model approach is also in line with The International Private Equity and Venture Capital Valuation (or IPEV) guidelines on fair value (or FV) for unlisted assets, where FV is the estimated price at which an asset can be potentially bought or sold in the open market. Accounting standards such as IFRS (International Financial Reporting Standards) or US GAAP (Generally Accepted Accounting Principles), both support the accounting of financial investments at their FV. Furthermore, the publication of IFRS 9 in 2014 removed the option to use historical cost accounting for financial assets, thereby making marking to market essential.

Market participants naively justify not using FV in their valuations of private companies by arguing that these assets are held for a longer term or till maturity, and hence there is no need to incorporate all information. By definition, FV does not permit any consideration of the holding period; instead, it requires an estimation of a value as if the market exists, even in the absence of such a market, thereby requiring any estimation to favour observable market values. Thus, a factor model approach calibrated with observed transactions is the most compatible method to compute FV for private companies.

Furthermore, regulatory changes are happening in private markets that further reinforce the need to improve valuations. In August 2023, the Securities and Exchange Commission (SEC) in the US put forth new regulations that apply to private market funds in order to improve disclosures, auditing, preferential treatment for some limited partners, and valuation opinion requirements for funds (SEC, 2023).

Following this, in September 2023, the Financial Conduct Authority (FCA) in the UK proposed a review of valuation practices in private markets, specifically focusing on their discipline and governance (Noona, 2023). Although these regulatory interventions do not seem to be well received by funds on the grounds of escalating compliance costs and potential unintended consequences, these changes do highlight the increasing importance of private markets and the need to improve valuation and disclosure practices.

The rest of this summary provides more details on the factor model approach and key findings:

Factor models have been widely used to quantify the risk factors that affect the value of an investment and to measure the sensitivity of each investable asset to these factors. When applied to listed stocks, factor models capture the systematic component of risks well, with the idea being that any non-systematic (or firm-specific or idiosyncratic) component of risk cancels out in a well-diversified portfolio, and hence should not have any effect on returns.

Even among private companies, General Partners (or GPs) explicitly or implicitly evaluate investments on company characteristics, market conditions, and deal characteristics based on their belief regarding how such characteristics have shaped performance in the past. Thus, it is feasible for an appropriately specified factor model to adequately capture the systematic component of private company valuations, especially at aggregated levels (e.g., segments) where idiosyncratic factors may cancel one another out.

The factor model is constructed using data on actual transactions (or investments) in private companies rather than appraised or estimated valuations. The appraised valuation of a private company, especially when a PE investor is a shareholder, is a severely biased estimate of its true valuation. Investors agree that such valuations are unrealistically "smoothed" and do not reflect all available information. Returns computed on such a biased valuation are unlikely to represent the true return generated by a private company, making them unusable in a factor model.

Moreover, the focus in this paper is on the total private company valuation (**P/S** ratio) explained by the factor model rather than a pro-rated investment in any of the company's security in the capital structure (e.g., investment holdings) or fund valuations (e.g., private equity Net Asset Value). This choice is because the starting point for the valuation of any security is the company itself, and it is the company's performance that fundamentally affects the valuation of different securities in its capital structure.

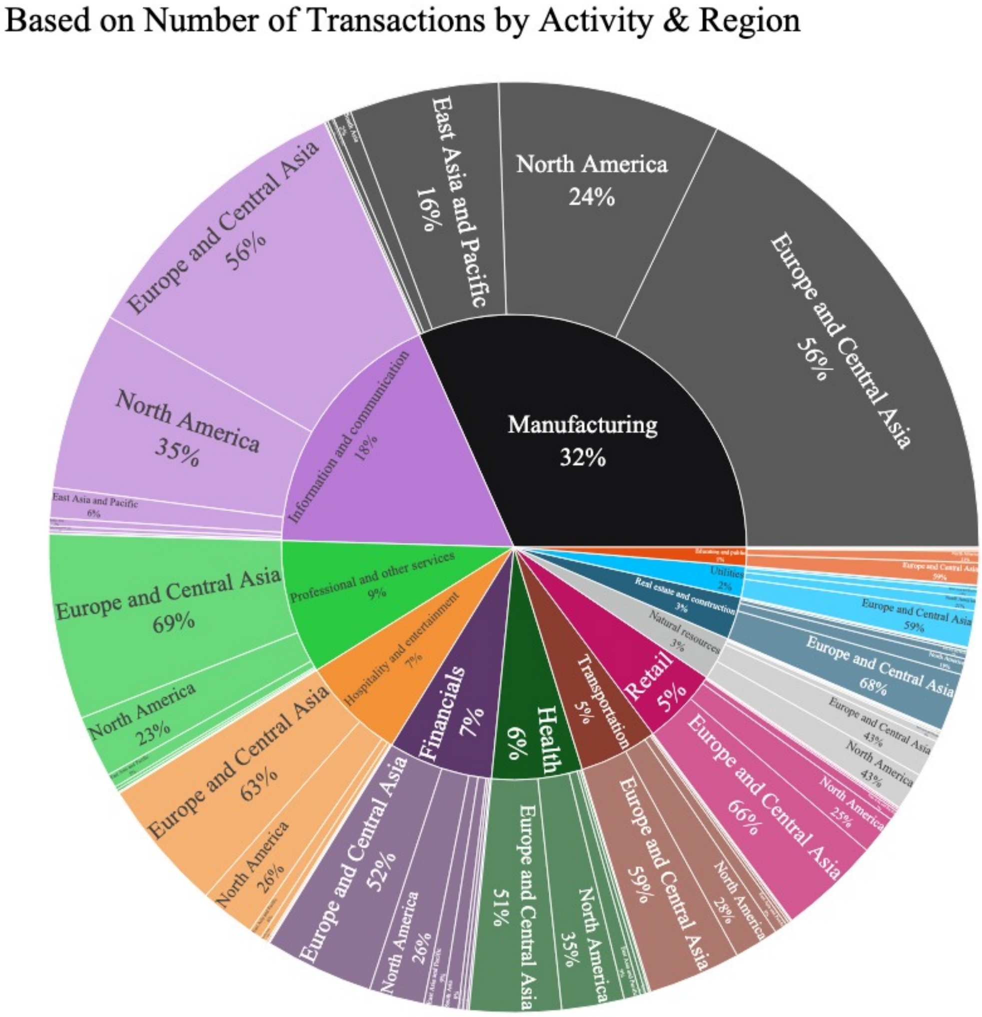
The factor model is calibrated using a global sample of transactions in private companies, one that is large, representative, covers all the sectors and spans a period of more than 20 years. The distribution of the sample is presented in Figure 1 according to the industrial sector and geographic region.

The proposed factors are based on prior academic work, a survey of private equity fund managers, private-market-specific characteristics, and PECCS™. Exploring several potential factors, the final list of factors is selected by adopting econometric approaches that trade off model complexity versus accounting for the observed variation in the sample. The chosen factors and their average effect on **P/S** ratio are shown in Table 1. Nonlinear factors in the model are omitted from the table for simplicity.

The statistically significant predictors of **P/S** in the model indicate that smaller, profitable, more leveraged, labour-intensive, innovative, and young firms command a higher valuation. Also, companies receive a lower valuation when the transaction is structured as an add-on (i.e., a portfolio company acquires a related private company). Moreover, transactions are more valuable when the market or industry valuations in public markets are high, term spread is lower, public market liquidity is higher, and when value stocks (high book-to-market ratios) receive higher returns than growth stocks in public markets. Finally, private companies operating in the financials, health, natural resources, and real estate sectors command a valuation premium whereas those that operate in the retail sector command a discount. Similarly, private companies that follow a subscription revenue model, or sell their output to end-consumers (or individuals), and whose output combines both products and services experience higher valuation. These documented effects of predictors on the valuation are also time-varying.

Diagnostic tests of the model performance indicate that the average predictions are very close to the observed transactions, and the errors in predicted values of valuation in in- and out-of-sample tests are very close to zero and follow a normal distribution. Out-of-sample tests are performed by randomly splitting the sample into two parts and examining how well the model can predict **P/S** ratio in the held-out sample. Even within each PECCS™ class, the errors are found to be very small, indicating the model can proxy segment-level valuation very well.

Figure 1: Sample distribution of transactions



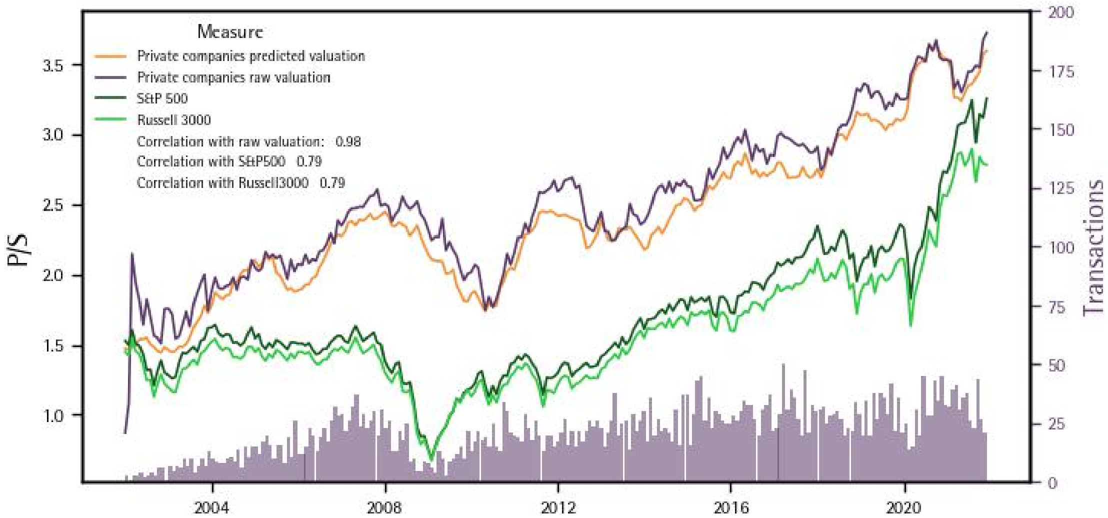

The trends in predicted valuations from the model, presented in Figure 2, indicate the following: First, the 12-month moving average of the model predicted **P/S** is desmoothed and exhibits comparable levels of volatility with public equity benchmarks. Second, the predicted **P/S** time series is highly correlated with public equity benchmarks. Third, the predicted **P/S** is remarkably similar to the 12-month moving average of raw transaction valuations, alleviating the concern that the model introduces any unnatural variation.

In conclusion, in this paper, we construct a robust factor model to explain the valuation of private companies. When combined with a novel taxonomy for private companies, PECCS™, this model enables the production of robust segment-level valuation metrics. Specifically, the factor model can be applied to a large set of private companies' financials and other information to obtain their shadow prices; these can form the inputs for constructing robust, granular, and precise benchmarks on a highly frequent basis. Such benchmarks will produce mark-to-market valuations for private companies through both the evolution in factor prices (which can be obtained by model calibration to new transactions) and changes in factors (e.g., characteristics, market, and macroeconomic variables). Such an approach overcomes the typical problems presented by current data in private markets such as subjectivity, behavioural biases, and data sparseness.

Table 1: Final factors in the model

| Factor | Source | Avg. effect on P/S during 1999-2022 |
|---|---|---|
| | Private Company Characteristics | |
| Size | Prior research & survey | Negative |
| Growth | Prior research & survey | Indeterminate |
| Profitability | Prior research & survey | Positive |
| Leverage | Prior research & survey | Positive |
| Labour intensity | Prior research | Positive |
| Patent | Prior research & survey | Positive |
| Hitech | Prior research & survey | Positive |
| Age | Prior research & survey | Negative |
| | Transaction Characteristics | |
| Addon | Private market features | Negative |
| Control | Private market features | Negative |
| | Market Characteristics | |
| Market valuation | Prior research & survey | Positive |
| Term spread | Prior research & survey | Negative |
| Industry valuation | Prior research & survey | Positive |
| Market price impact | Prior research & survey | Positive |
| Value factor | Prior research | Negative |
| | PECCS™Pillars | |
| PECCS classes | Prior research, private market features, & survey | Depends on each class |

Figure 2: Trends in model predicted, raw, and public market valuations



# 1. Introduction

In this paper, we develop a factor model to explain the observed transaction prices of private companies. Relying on a global dataset of private company transactions spanning more than 20 years, factors that influence the valuation of private companies are proposed, identified, and validated.

## 1.1 Private Assets Landscape

McKinsey (2023)'s Private Markets Review estimates the size of private markets to be $11.7 trillion, viz-a-viz public equities that constitute $105.1 trillion (Statista, 2022). Private equity (or *PE*) remains the majority investment vehicle to access these private companies and by assets under management (AUM) accounts for 65.1% of the total AUM in private assets (43.1% if venture capital is excluded from PE). By sheer size, private assets form an important part of financial markets. Moreover, by their contribution to GDP, employment, innovation, and even pollution, private companies form a non-trivial and pivotal part of the global economy. [1]

### 1.1.1 Gaps in Valuation

The valuation of private companies is critical, given its role in portfolio screening, allocation, monitoring, and even compensation decisions. For example, asset owners often use charts of the correlation between the performance of different asset classes in adjusting their allocation decisions. But the comparison for private assets is generally carried out on the basis of appraised valuations that incorporate unrealised profits and thus are very subjective.

1 - According to the American Investment Council, about 6.5% and 7.0% of US GDP and employment, respectively, are attributable to private equity (Morgenson, 2021). Also, PE ownership increases patents by 6.0% (Amess et al., 2016). Furthermore, the PE Stakeholder Project (PESP) documented that, since 2010, PE funds have invested more than US$1.1 trillion in pollution intensive energy assets (Reclaimfinance, 2022).

Similarly, carried interest – the performance fee component of fund managers – relies on the valuation of assets. Cross-collateralised waterfall structures, increasingly popular outside of Europe, allocate carried interest based on realised and unrealised profits, making valuation an essential component of performance based compensation (Stefanova, 2017).

Private companies are largely open only to institutional money and hence in general have sophisticated investors. Even so, because of its size, role in the economy, and potential spillovers to main street financial markets, the valuation of this asset class should be a key priority to a broader audience. Currently, even investors face a dearth of data to help them arrive at the fair value of their investments in private assets. They often must solely rely on GPs (i.e., general partners or fund managers) and associated service providers to appraise difficult-to-value, less liquid, and often not recently traded private companies.

Key drawbacks in the valuation of private companies include the lack of timely incorporation of available information into valuation and the lack of comparable performance metrics (Gompers et al., 2016):

**First**, the valuations are not marked-to-market. Typically, GPs report the valuation of their portfolio holdings quarterly to their investors (i.e., limited partners or LPs). Although accounting standards lay out the principles to value these illiquid assets, the guidelines are broad and offer multiple methodological choices. Consequently, the valuation is strongly influenced by the GPs and is subject to scrutiny only in the annual audits or when a potential buyer is examining the asset, which can typically be at least five years after acquisition. Although fund contracts do require GPs to exercise fiduciary duties to the LPs (e.g., Wikowsky, 2020), the lagged reporting, annual management fee structure, the committed nature of the capital, and concentrated ownership do not incentivise the GPs commensurately to mark their investments to market.

**Second**, the illiquidity of the private companies, and the associated difficulty in computing returns and volatility, have created distortions in examining the performance of private companies. These include a reliance on internal rate of returns or IRRs, which can be gamed and differ very much from LPs' realised returns.

### 1.1.2 Why Fair Value cannot be Ignored?

A naive justification for not considering fair value (or FV), or in other words delaying/ignoring marking to market the valuation of private companies, is that these assets are held for a longer term or until maturity. Hence, the argument is that there is no need to adjust valuation according to variations in factor prices.

However, this approach of ignoring FV is not compatible with the application of accounting standards such as IFRS (International Financial Reporting Standards) or US GAAP (Generally Accepted Accounting Principles), both of which support the accounting of financial investments at their FV. Moreover, the publication of IFRS 9 in 2014 has decreased the potential to use historical cost accounting for financial assets, making marking to market essential.

FV is the exit value at the time of consideration and, by definition, does not permit any consideration of the holding period to be incorporated. Thus the principle of FV is that, in the absence of a market, it is possible to estimate a value as if the market exists. Any estimation should therefore constitute a proxy for this market value and, as such, the modelling should always try to favour observable market values. Also, irrespective of the methodology used to arrive at FV, there can only be one FV per asset at a time, and this unique FV cannot depend on the investor's holding intentions or business model.

The International Private Equity and Venture Capital Valuation (or *IPEV*) provides guidelines for private market participants, and its board's view of FV is in alignment with US GAAP and IFRS. For example, IPEV (2022) guidelines state:

*"The Valuation Guidelines have been prepared with the goal that Fair Value measurements derived when using these guidelines are compliant with both International Financial Reporting Standards (IFRS) and United States Generally Accepted Accounting Principles (US GAAP). This has been done in order to provide a framework, which is consistent with accounting principles, that Private Capital Funds should utilise to determine a Fair Value for Investments."*

IPEV reiterates that FV can only be a value that corresponds to the market parameters at each measurement date; this definition leaves no room for the idea of a conservative principle such as claiming that the valuation is done below the FV at exit value or that exit usually happens at a higher price. Such claims are often promoted by GPs as providing a smoothing effect and protective effect on PE investments; they are in violation of FV principles and do not give investors better information about their investment performance or future potential.

Even in the US, regulations such as Accounting Standards Codification 820 (ASC820),[2] enacted

2 - Before ASC820, US GAAP allowed PE funds to report valuations based on historical costs or simply the valuation in the latest financing round (Easton et al., 2020). ASC 820 defines how FV should be calculated for financial reporting purposes. It estab-

in 2018, requires PE funds to report the FV of the assets to their investors.

ASC 820 lays out a three-level hierarchy of FV measurements, with level one for liquid assets, level two for assets that can be priced with market inputs such as derivatives, and level three for highly illiquid assets, such as investments in private companies. IFRS 13 applicable in Europe also provides similar recommendations for computing FV of illiquid assets (e.g., Grant Thornton, 2021).

Thus, using the proposed factor model is in line with IPEV guidelines, ASC 820 framework, and IFRS 13 for unlisted assets. Specifically, calibrating the model with observed transactions that are orderly transactions happening at arm's length automatically adheres to the recommendations on *calibration* and *backtesting*, respectively, in IPEV guidelines.

### 1.1.3 Implications of the Valuation Gaps

These valuation related issues are not mere theoretical expositions but have several real-world consequences, some of which are detailed below:

1. **Return smoothing and burnished performance:** GPs generally take much longer to incorporate market information into reported valuations, leading to the phenomenon of smoothed returns (e.g., Crystalfunds, 2022). Reporting infrequently and in a lagged manner mechanically produces smooth returns even without considering GPs' incentives to delay incorporating market information into their quarterly valuations.[3] The flip side of these smoothed returns is that the performance looks burnished or too good to be true as these assets appear to produce reliable results with less risk, leading to misunderstanding for allocators (e.g., Armstrong, 2021). An illustration of this phenomenon is provided in the next chapter.
2. **Ephemeral down rounds and structures:** PitchBook estimates from 2015 to 2022, more than 70% of VC rounds are at a higher valuation, while an equal proportion of the remaining financing is flat or down rounds (Temkin, 2022). This reluctance to mark down the valuation of investments is particularly detrimental to private companies looking to raise further capital, such as for growth opportunities, thus starving them of financing and potentially leading to risky management practices.
   Similarly, the reluctance to raise capital in a down round can also lead to unnecessary deal structures. For example, BlackStone Real Estate Investment Trust (or *BREIT*), a non-publicly traded REIT with $69 billion in AUM that relies on reported valuations, offered redemptions of up to 5% per quarter (e.g., Bary, 2023). Faced with rising redemption requests in 2023, BREIT solicited investment from the University of California as LP to the tune of $4 billion at the reported NAV, but with additional benefits over other investors in the form of an 11.25% preferred return buttressed by a backstopped margin of $1 billion in BREIT shares (e.g., NAREIT, 2023). So much complexity seems to be added to the fund to avoid a simple markdown in the reported valuation. Although this paper's focus is not on private real estate, this example illustrates what distortions reported valuations can produce, and BREIT exemplifies the reluctance to mark down arguably simpler assets such as real estate. This example also indicates that other growth oriented private companies can have more severe valuation problems.
3. **Denominator effect:** An ironic outcome of this valuation malaise is the "dreaded denominator effect" for LPs with strict

lishes a FV framework applicable to all measurements under US GAAP.

3 - Periods of high volatility in main street financial markets, such as in 2022, increase the attention on alternative investments such as PE, where valuations seem robust in comparison. The comparatively high and stable valuations of private companies can and do distort asset allocations, and are likely to have long-term consequences for the asset class and its future performance.

thresholds for their private asset allocations. When public financial markets perform poorly, their valuations immediately fall, decreasing their weights in a diversified portfolio, while private assets hold their value. This breaches the absolute asset class thresholds set by LPs, thereby mechanically reducing new capital entering private companies and thus not allowing investors to diversify across vintages. For example, in 2022, several US pension funds had reached their targeted private allocations and suspended further investment programmes (Amenc and Blanc-Brude, 2023). Moreover, being overweight can also create fire sales in the secondaries market at depressed prices and create LP requests to stop capital calls (e.g., Weinberg, 2022).

4. **Diverging secondary market valuations:** The fall in the number of public listings and publicly traded firms (e.g., Chemmanur et al., 2022) and the rising allocation to private markets (e.g., Shen et al., 2021) enable private companies to stay private much longer or indeed forever. This contrasts with the typical GP fund life of 10 years, thus necessitating the need for GP exits, giving rise to this increasingly growing category of GP-led secondaries (Lussier and Biamonte, 2022). In addition, the typical private fund lock-in period of 10 years has created significant demand for LP liquidity, giving rise to an alternate secondary market that supports LP exits but which, however, is frowned upon by GPs (Thomas, 2022).

   These two secondary markets provide a real-world test of the reported valuations. What has been observed from the limited data on these two markets is that the GP-led secondaries, like continuation funds (i.e., GP remains the same) with some exiting and new investors, happen at a valuation similar to the quarterly reported NAVs (e.g., Hamlin, 2022). However, in LP secondaries, which represent clear arm's-length transactions, trades happen at steep discounts (e.g., Farman, 2022).

All these phenomena risk providing incorrect risk-return picture to asset allocators, leading to inefficient capital allocation and distortions, preventing LPs from making the most optimal decisions. Moreover, when GPs are diversifying their investor base and seeking capital from various other investors such as defined contribution (DC) plans and retail investors (e.g., private equity democratisation (Mendoza, 2022)), the valuation of private companies needs to be more frequent and precise.

### 1.1.4 Are Public Benchmarks the Solution?

Private companies are inextricably linked to the economy as they must face market demand, obtain financing, withstand business cycle fluctuations, and access capital markets for investor exits, thus exposing them to the same factors as publicly listed firms. So a casual view can indicate public markets as being appropriate benchmarks for private companies. However, that view would be incorrect for the following reasons:

1. **Disappearing US public firms:** Since the 1997 peak in public listings, the universe of listed firms in the US has fallen every year since then. In addition, share buybacks have outweighed share issuances even among listed firms (Doidge et al., 2018), leading to greater capital outflows than inflows into the public markets. Several factors underpin this trend, such as increasing intangible capital stock that makes publicly listed firms more vulnerable due to disclosure requirements. Additionally, the post-SOX demands of staying public have increased compliance costs for smaller public firms. Finally, the growth in private capital has also contributed to this trend, by allowing companies to stay private for longer or forever. Such concentration of stocks in the US public markets has reduced their diversity in terms of growth, size, and indus-

trial sectors. For example, the technology sector comprises 28% of S&P 500 in April 2022 (Wang and Pacheco, 2022), before the index provider decided to reclassify some stocks into other sectors.4

Thus, overall, there is a key selection mechanism in the decision to stay public, depending on the country of the asset, thus giving rise to systematic differences: large and old public US firms stay public while young and R&D-intensive companies prefer to stay private. Additionally, the rising tide of regulations regarding ESG disclosures by public companies may accelerate delistings and make private firms fundamentally different from publicly listed companies.

Thus, market indices featuring US stocks prominently, and even global indices (e.g., the weight of US stocks in MSCI World Index is 69.5% in 2022), and consequently public market factors based on these indices, may provide poor proxies or insufficient proxies for private companies.

2. **Leverage:** Leveraged buyouts, a key strategy of PE investment, rely on leverage at the asset level to generate value. Therefore, typical leverage levels in PE-owned private companies are higher than in public stocks, especially during a period of benign interest rates such as between 2000 and 2020. Confirming this view, studies have found that investing in highly leveraged small-cap stocks produces similar returns to private equity (Chingono and Rasmussen, 2015). Thus, using public indices as benchmarks could misrepresent risk for leveraged private companies.
3. **Diversification:** The costs of diversifying a portfolio of stocks to investors are relatively lower than executing a diversified strategy among private companies.

4 - This trend of vanishing public listings has not been borne out in the rest of the world, which has witnessed a modest uptick in the number of listed companies, at least in the most recent years (e.g., WFE Research, 2022). However, this trend is not driven by a decrease in dual listings or foreign firms in the US, as that number has remained stable in the last two decades (e.g. Brorsen, 2017).

In private companies, the higher due diligence requirements, increased operational involvement, or in general the limits on GPs' time and resources, have led to a design where GPs have a smaller breadth of holdings in private companies to apply strict and intensive governance.

Although investing through fund-of-funds can enable LPs to diversify, such investment vehicles add another layer of fees on top of the individual fund's fee structures. Moreover, investing in several private equity funds directly to diversify can lead the private capital investment programme to increase outlays substantially. Such limitations to diversify make LPs selective in their approach and engage in careful risk assessment through due diligence.

Thus, public benchmarks that are diversified may make poor proxies for concentrated portfolios of private companies.

## 1.2 The Proposed Approach

Although FV guidelines proposed by standards such as IFRS and US GAAP are well intentioned, these principle-based guidelines become less appropriate in practice, especially with limited data. For example, ad-hoc approaches to compare multiples of similar companies based on recent transactions lack formality and are subject to biases such as the staleness and sparsity of comparable values and subjectivity in selecting peer groups.

Thus, we propose a *factor model* of prices that can be estimated on a sample of observed transactions to obtain unbiased factor price estimates. Factor prices refer to the premium (or discount) that an investor is willing to pay to seek exposure to a specific factor of return in private companies. For example, observing the relationship between size and valuation among reported transactions, it can be inferred how much premium or discount an investor is willing to pay for purchasing a larger private company.

An important and key application of this approach is that, with the estimated factor prices, say for size, it would then be possible to price unlisted private companies whose size information is available, irrespective of whether they are traded or not. This approach provides a more robust estimate for FV and enables the creation of representative indices of private companies.

In the subsequent sections, we use a large sample of observed PE investments in private companies since 1999 to estimate the effect of several potential factors on valuation. Starting with factors based on prior well-established academic work on listed securities and those based on the unique institutional features of private capital markets, the optimal set of factors is chosen to fit the data well. We identify a sparse set of factors, making use of the latest advances in econometric approaches (e.g., forward stepwise selection and Lasso regressions).

Additionally, we estimate the factor prices in a time-varying manner, using a state space model (also known as a dynamic linear model). An advantage of this approach is that it enables robust estimation of unbiased factor prices, or in other words hidden values, even when the observed data is serially correlated, biased in terms of time and sectors, and noisy due to the idiosyncratic characteristics of each transaction. For example, deals in the technology sector might be clustered in certain time periods or countries, and a specific transaction price may be influenced by the individuals involved, thus adding noise to the observed price. A state space model calibrated using the sample can infer the unbiased factor price while ignoring the noise component inherent in each price.

The rest of this paper is organised as follows: Chapter 2 discusses why a model based approach is the first-best solution for this valuation problem and also explores the flaws in current approaches to valuation and performance evaluation. Chapter 3 summarises the potential factors. Chapter 4 discusses the data while Chapter 5 discusses the empirical approach. Chapter 6 focuses on selecting a subset of optimal factors that determine private company valuations, estimating ordinary least squares, and dynamic linear model regressions of valuation, and discusses those results.

Chapter 7 presents robustness tests of the model and also reports the segment level trends in valuation during the sample period. The trends are also contrasted with public equities, and unsurprisingly, the valuations in both markets tend to be strongly correlated. Moreover, the trends in valuation also illustrate the heterogeneity in private asset performance across PECCS™ segments.

Chapter 8 proposes other applications of the factor model and concludes.

# 2. Private Market Valuation is Always About Models

In public markets, recent transaction prices make it easy for investors to assess the performance of their holdings. Also, their portfolio value measured at any point in time is close to what could be realised if they were to exit immediately, adjusting for liquidity. However, in private markets, recent transaction prices are not observable and in addition reported valuations are not close to what could be realised in a fire sale.

Thus, the only way to ascertain the valuation of private assets is to use models. Even if one disagrees with relying on a model, in the absence of market prices, any valuation technique is still going to implicitly rely on a framework and assumptions – or in other words a model. For example, both market-based methods (e.g., comparables) and income-based methods (e.g., discounted cash flow or DCF) are all, in the end, model based and require both multiple assumptions and a framework. Therefore, to compare valuation approaches, it is important to understand what makes a *good model*. A *good model* can transform sparse, biased, noisy, and limited data in private markets into useful information that can aid investors.

In this chapter, after discussing the criteria of a good model, several alternatives are compared on these metrics. Next, we provide a critique of existing approaches through examples, followed by reasoning on how to improve the models. Finally, we elucidate the advantages of using a good model, and specifically the model proposed in this paper.

## 2.1 What Makes a "Good" Model

Broadly characteristics of a good model can be grouped into two categories of *formal* and *technical*, discussed further below.

### 2.1.1 Formal Characteristics

- **Theory-based**: Good models need to be grounded in formal theory rather than being ad-hoc or practitioner based. For example, DCF approaches are based on the simple theory of the time-value of money and risk-return tradeoffs (Damodaran, 2007).
- **Arbitrage free**: Valuation models also need to be based on the principle of arbitrage free pricing, i.e., prices adjust until there are no opportunities left for riskless profits. For example, the DCF approach is arbitrage free as it equates the value of an asset to the present value of its expected future CFs.
- **Consistent with accounting standards**: Several accounting bodies and industry organisations have guidelines in place for the valuation of private companies. For example, in the US, Accounting Standards Codification 820 lays out a three-level hierarchy of FV measurements. Also, IPEV guidelines include comprehensive guidance on private asset valuation (e.g., IPEV, 2022). Thus, a proposed model should be consistent with such guidelines on valuation.
- **Taxonomy framework**: Calibrating a model and its application requires a taxonomy framework for segmenting private companies. Any taxonomy should be able to group similar assets and account for their systematic risks. For example, in comps, an industry definition is central

to identifying peers and average price multiples. More accurate and granular industry definitions can give more precision to valuation.

#### 2.1.2 Technical Characteristics

- **Robust**: A good model will have smaller pricing errors both in-sample (training) and out-of-sample (prediction), where error is defined as the difference between the estimated and actual value. Good models, also, need to be robust, replicable, and produce the same estimates with identical inputs.
- **Explicit**: A good model needs to be clearly described, documented, and verifiable – or in other words be an explicit model. Moreover, the inputs to the model should be measurable for all assets. This ensures the model can be used to estimate the valuation of any asset objectively.
- **Misspecification**: When an irrelevant independent variable is included in the model, it can lead to over- or under-specification (e.g., omitted variable bias) issues, resulting in biased and inconsistent estimates (e.g., Chau and Chin, 2003). However, misspecification to some extent is unavoidable in private markets due to the unavailability of reliable or complete information. Thus, a good model needs to be parsimonious and maximise the inferences drawn about the valuation of private companies with the limited data available.
- **Predictive**: A fundamental purpose of valuation is to provide an estimate when one cannot be observed. Thus, a good model also needs to be predictive, i.e., with observable inputs should generate a valuation estimate. In the asset pricing context, it means that both factor loadings (or $\beta$'s) and factor prices (or factor premiums) need to be computable to predict the valuation.
- **Frequency**: A good model should be able to estimate valuation on a highly frequent basis. This also means the inputs used in the model need to be frequently observable. For example, comps based on recent transactions can only be performed infrequently unless regular transactions in similar companies are observable.

Based on these criteria, the two most commonly used approaches to valuation: the DCF and comps methods are evaluated along with the proposed factor model in Table 2. The proposed factor model is better or as good as each of these alternatives in all the criteria. Thus, the application of this factor model to private markets is the first best solution to solving the current data problems. More details on how each of these features is addressed by the proposed model are explained in the subsequent chapters.

### 2.2 Limitations of Common Industry Practices

Appraisals refer to an assessment of the fair market value or FV of a company, and participants in private markets use various methods for their appraisals. Once a valuation is determined, there are also different methods to report performance. Note that performance measurement requires at least two observations of transaction price or estimated valuations or one of each, whereas valuation requires a single estimate. In this section, limitations of common industry practices to estimate valuation and report performance are described.

#### 2.2.1 Valuation Approaches

*Market-based Approaches*

Market-based approaches to valuation rely on using market related inputs such as the valuation of similar publicly listed peers or recent transactions to arrive at an estimated valuation. Comps analysis is a very commonly used market-based approach to apply a listed sector or listed/unlisted peers' price multiples

Table 2: Comparing valuation models

| Model ▶ Features ▼ | Discounted cash flow | Comps analysis | Our factor model |
|---|---|---|---|
| **Formal characteristics** | | | |
| Theory | 1) Time value of money<br>2) Return-risk tradeoff | Rule of thumb | 1) Time value of money<br>2) Return-risk tradeoff<br>3) Asset pricing theory<br>4) Hedonic pricing |
| Arbitrage free | If calibrated well, produces one price | Multiple prices supported by adjustments required | Produces one price at a time based on factor price and factor evolution |
| Accounting standards | Aligned with illiquid assets guidance | Aligned with illiquid assets guidance | Alignment with market inputs guidance (better) as uses transaction data to calibrate the model. |
| Taxonomy | 1) Needs taxonomy to estimate CFs or discount rate<br>2) Flexibility in choice causes disagreement | 1) Needs taxonomy to select peers<br>2) Flexibility in choice causes disagreement | 1) Rigorous, objective PECCS™ framework<br>2) Captures several non-industry risk factors |
| **Technical characteristics** | | | |
| Robust | Flexibility in inputs can cause huge errors | Flexibility in inputs can cause huge errors | 1) Robust, accurate, & granular as inputs cannot be selectively used<br>2) Small segment level errors |
| Explicit | Input choices unclear & not explicit | Peer/transaction selection unclear & not explicit | Model described clearly, documented, & verifiable |
| Misspecification | CF or discount rate choices suffer omitted variable bias | Unaccounted difference with peers causes omitted variable bias | Parsimonious |
| Predictive | Not predictive, as one company's DCF is not useful for another | Yes, price multiples can be used | Yes, can produce ex-ante valuation measures |
| Frequency | Low frequency | High frequency for public peers but low frequency for recent transactions | High frequency can even be performed monthly or daily |

(e.g., P/S, P/Ebitda, or EV/Ebitda) to a focal private company to arrive at its valuation.

Comps analysis can be visualised as a simplified form of the dividend discount model (DDM). The traditional single-period model can be stated as in Equation 2.1 where $P$ is the valuation, $D_1$ is the dividend next period, and $r$ and $g$ are the required return and sustainable growth rate.

$$\begin{aligned} P &= \frac{D_1}{r-g} \\ P &= \frac{E_1 \times DPR}{r-g} \\ P &= \frac{S_1 \times Profit\ margin_1 \times DPR}{r-g} \\ P/S &= \frac{Profit\ margin_1 \times DPR}{r-g} \end{aligned} \tag{2.1}$$

The dividend can further be expressed as a product of an earnings measure $E_1$ (e.g., net income or free cash flow) and the payout ratio $DPR$. $E_1$ further can be expressed as a product of the sales in the next year $S_1$ and the profit margin. Rearranging provides an expression for **P/S** ratio in terms of DDM inputs.

When a peer's **P/S** is projected on a company, it is comparable to assuming that the two companies have similar profit margins, retention rates, required returns, and growth rates in the DDM framework. Thus, it is possible to imagine the comps analysis to be an implicit dividend discount model.

In the end, comps analysis is a very powerful method to perform valuation, as it relies on the closest observable proxy to arrive at the valuation. However, the way it is applied in practice, especially in terms of the quality and quantity of inputs, is problematic, as described below:

- **Known differences between the company and its peers**: The implicit assumption of comps is that the focal private company faces identical risk factors and identical sensitivities to those risk factors as the peer(s). However, there could be known systematic differences between the companies which are not accounted for when using its multiple. Subjective adjustments for known differences, discussed separately, create further problems.
- **Low quantity of inputs used**: Only price multiples are used when more information about the focal company and its peers is available.
- **Low quality of inputs**: Use of both handpicked peers and a custom list of transactions allows flexibility in choice to present a valuation as one pleases. Furthermore, when using transaction data, due to the unavailability of a large sample, stale transactions may be included further reducing the utility of this approach.
- **Adhoc adjustments**: Sometimes, to account for differences between a company and its peer, say on account of illiquidity, size, leverage, etc., practitioners may introduce adhoc adjustments to the price multiples. These adjustments are informal and subjective and not driven by theory or data, thus making the valuation uninformative and possibly misleading decision making for investors.

*Income-based Approaches*

Income-based approaches to valuation rely on using past or expected cash flows from the company adjusted for the level of uncertainty in realising these cash flows. A popular income-based approach to valuations is using the DCF method. DCF uses a simple model of discounting future cash flows and needs expected cash flows, discount rates, and terminal values as inputs. Since private companies can be regarded as going concerns, terminal value computations can be further simplified based on Gordon's growth model (or DDM) assuming steady growth at terminal horizons. Thus, applications additionally require a horizon and terminal growth rates.

DCF approach is a strong tool to estimate valuation, grounded in theory, and simplistic to apply. However, in practice, some problems emerge in applications of DCF for the valuation of private companies including:

- **Highly specific inputs**: Most of the inputs to DCF are company specific. Like for example, DCF analysis requires modelling of future revenue, expenses, depreciation, and investment to arrive at free cash flows. Such intensive and uncertain input requirements provide a high level of flexibility for the one using DCF. Often in practice, it is possible to work your way backward from a valuation to choosing a discount rate and cash flow streams that seem reasonable, defeating the purpose of coming up with an objective valuation.
- **Incorrect discount rates**: Often in DCF analysis of private companies, fund managers use each fund's target IRR as discount rates. This is highly inappropriate as the discount rate is supposed to account for each company's risk and cannot be based on a one-size-fits-all approach.

*Frequency of Valuations*

Apart from the valuation approaches, it is also important at what frequency the valuation is performed. Currently, valuation is performed at a low frequency that is uninformative for asset allocation strategies. Low-frequency valuations might be a symptom of a combination of factors including choosing the valuation method and the associated input and skill requirements to perform it. For example, thirdparty fairness opinions on valuations can cost GPs up to $300,000 per opinion (Dayal, 2023), indicating that this could be a fairly expensive exercise if done frequently involving external parties. Moreover, lack of regulation and LP tolerance leads to most funds estimating valuations at a quarterly or semi-annual frequency.

However, investors not knowing the valuation frequently does not decrease the inherent risk in an asset class, protect the investor, or help their decision-making. On the contrary, low-frequency valuations paint an unrealistic picture of the risk of investing in private markets, showing them as less volatile, safer, and stable investments. Thus, even if estimation problems are ignored for the moment, the frequency of valuations by themselves can produce distortions.

To illustrate this point further, in Figure 3, the returns and index levels of the S&P 500 are plotted at different frequencies. Specifically, they are plotted at monthly, quarterly, annual, once in two years, and once in four-year frequencies. In other words, if investors willingly stay uninformed of the S&P 500 performance at higher frequencies and only observe at a preferred lower frequency, then Figure 3 presents what they will observe. Additionally, to facilitate asset class choices, both the annualised Sharpe ratios and maximum drawdowns are included for each frequency.

If an asset allocator is asked to choose an asset class just using this data, there will likely be a tendency to choose the asset class represented by lower frequencies as it seems that they are less risky, have fewer periods of negative returns, and perform really well in some periods. These takeaways are also captured in their annualized Sharpe ratios and maximum drawdown measures, which are monotonically increasing and decreasing, respectively, as frequencies go down.

This is the typical issue with private markets when valuations are neither available frequently nor reliable, which understates the risk and burnishes performance. And this exercise illustrates clearly some investor's *perception of private markets*.

### 2.2.2 Performance Measurement

*Internal Rate of Returns*

Internal rates of returns or IRRs are widely used in fund reporting and benchmarking in private markets. The IRR is the discount rate at which the net present value of a series of cash flows (CF) equals zero, where CFs include inflows (investments) and outflows (distributions). IRRs can be computed at a holding level or fund level by GPs and are often disseminated as the performance metric of choice. Holding-level IRRs can also be aggregated into fund levels based on capitalisation weights.

Despite ita many shortcomings, the IRR is very appealing for private asset investments, as in theory it is based on actual realised CFs, and thus less subject to biases or assumptions. However, in practice, these considerations are likely trumped by the illiquidity of private investments and the discretion of GPs on the timing of cash flows (e.g., Phalippou, 2008).

- **Methodological concerns**: IRR suffers from multiple methodological problems, such as having multiple IRR solutions for the same CF stream and reinvestment rate assumptions that interim CFs can be reinvested at the same rate as the IRR. Methodological solutions exist to overcome each of these deficiencies (e.g., a modified IRR). However, they are not popularly adopted in private markets.
- **Combining estimated and realised CFs**: Due to illiquidity, in applications, realised CFs are combined with unrealised estimated valuations in IRR computations, thus making the metric less useful, as estimated valuations

Figure 3: S&P 500 observed at different frequencies

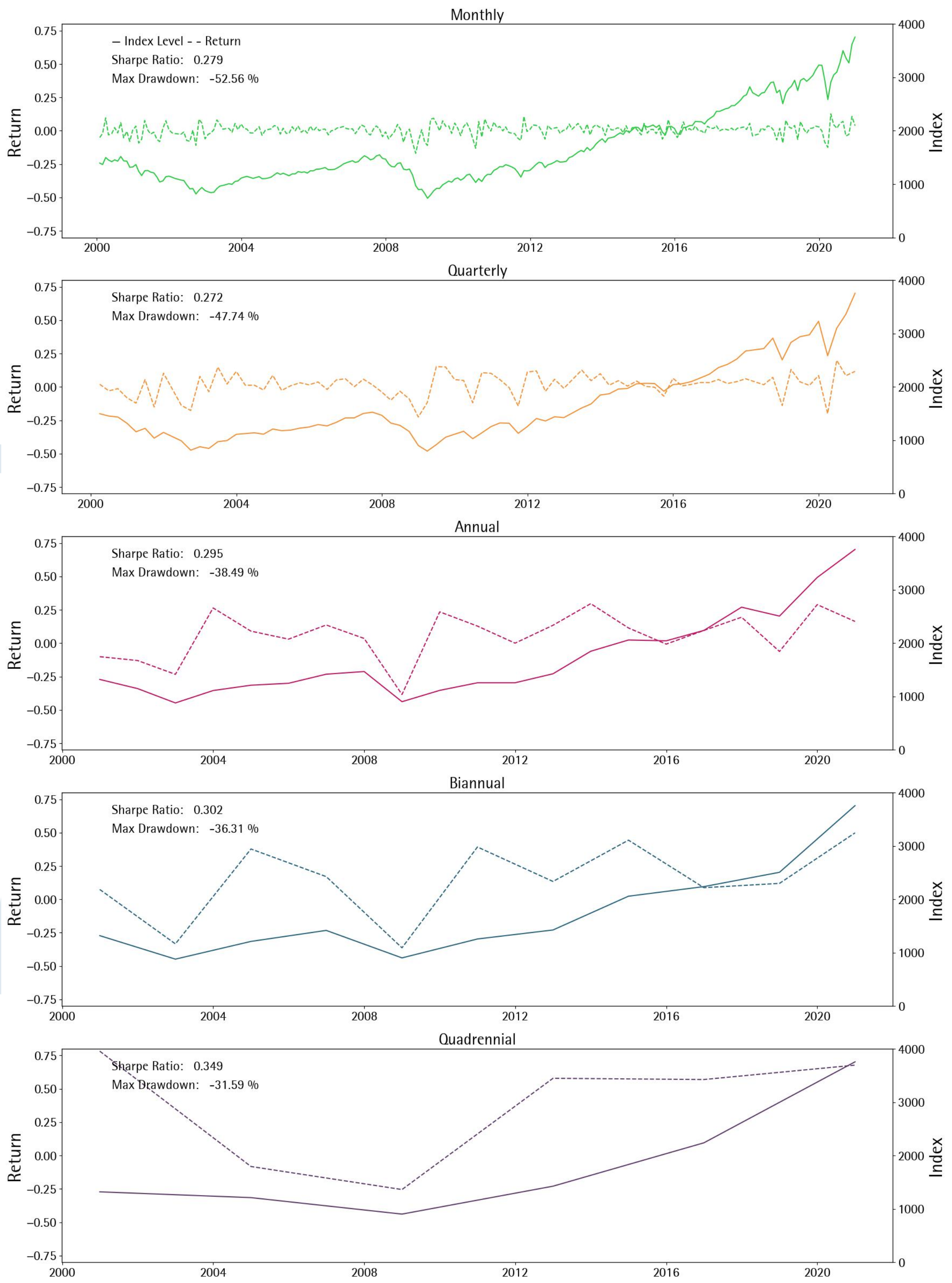



Table 3: Illustration of IRR anomalies

| Year | Fund A | Fund B |
|---|---|---|
| 2022 | -50.0 | -50.0 |
| 2023 | -25.0 | 100.0 |
| 2024 | 100.0 | -25.0 |
| 2025 | 50.0 | 50.0 |
| IRR | 41.42% | 100.00% |
| Time-weighted return | 51.83% | 51.83% |

are not standardised and can vary a lot based on methodological choices.

- **CF Timing**: IRRs are also distorted due to the timing of cash flows. As GPs have discretion on calling for capital and giving out distributions (at the fund level) or paying dividends and arranging debt financing (at the holding level), they can effectively game the IRR by their timing choices. In fact, many market phenomena indicate that GPs seek alternative financing to manage the timing of CFs, such as subscription lines or loans against commitments and net asset value-based loans. Such forms of financing can help manage IRR while increasing the costs for the fund.

Table 3 presents two examples of how the use of IRRs can distort private investments. Assuming there are two funds that call and distribute identical amounts of capital, but only differ in their timing as shown in Table 3.

The timing of capital calls and distributions severely distorts reported IRRs in this example. *Fund A* reports an IRR of 41% while *Fund B* which paid out earlier distributions reports an IRR of 100%. On the other hand, the LP's average realised returns differ significantly from these reported IRRs. For example, the time-weighted return, which is better in representing the average realised annual return (ignoring the irregular cash flows), works out to an annual 51.83% for both the above investments.

Thus, when GPs can influence the timing of these cash outflows and inflows, IRRs are subject to severe distortions, and lose meaning, especially when reported without the context of cash flow sizes and their timing.

*Public Market Equivalent Approaches*

The most commonly used private equity performance metrics include multiples of invested capital (MOIC) and IRRs. But both these metrics are inadequate at capturing fund performance and are not comparable across funds, the key requirement of any performance metric.

MOIC expresses fund performance as a multiple of its investment contributions, with greater values being preferable. A key issue of MOIC is that they do not consider the time value of money, the most important tenet in finance. Similarly, issues with IRRs are described above, specifically concerning how the timing and size of cash flows can distort them.

Some proponents propose a public market equivalent (or PME) approach to overcoming some of the problems associated with IRRs and MOIC. To take out the effect of timing from the performance metric, the most basic PME approach (e.g., Long Nickels PME approach) relies on building a portfolio that makes theoretical investments and withdrawals in a stock market index at the same time as the private equity fund calls for contributions or make distributions. Finally, the IRR of the fund is compared with the IRR of the theoretical portfolio (or PME) to get the IRR spread, i.e., how much the IRR exceeds the PME. Greater values of the IRR spread indicate the outperformance of the fund, and vice versa.

Several improvements have been proposed to the basic PME approach to tackle its deficiencies, but as illustrated below, is still unreliable.[1]

The PME approach in trying to solve one problem ends up creating another. In addition to all the flaws of IRR, PME that is based on IRRs can additionally rank funds incorrectly, when the market is volatile and funds differ on their timing skills. In the end, investors care about returns, and hence it may be inappro-

1 - Some of the improvements on PME includes the PME+ approach (Rouvinez, 2003) and the modified PME by Cambridge Associates that overcome the negative PME values when funds greatly outperform the markets. Also, Kaplan and Schoar (2005) computes the PME as a market multiple based on realised market returns as discount rates to apply to capital contributions and distributions of a fund.

Table 4: Flaws in PME approaches

| | | | Cash flows for | | Portfolio value of | |
|---|---|---|---|---|---|---|
| Year | Index | Index returns | Fund A | Fund B | Theoretical A | Theoretical B |
| 2022 | 100 | 0.0% | -75 | -75 | 75.00 | 75.00 |
| 2023 | 110 | 10.0% | -50 | 1 | 132.50 | 81.50 |
| 2024 | 160 | 45.5% | 1 | -50 | 191.73 | 168.55 |
| 2025 | 120 | -25.0% | 25 | 25 | 118.80 | 101.41 |
| 2026 | 120 | 0.0% | 115 | 115 | 118.80 | 101.41 |
| IRR | | | 3.60% | 4.07% | | |
| PME | | | | | 4.40% | 0.64% |
| IRR Spread | | | -0.80% | 3.43% | | |

priate to completely nullify managers' timing skills in their performance analysis.

Table 4 illustrates the computations for two *Funds A* and *B* that call and distribute capital from 2022 to 2025 and are valued in 2026. Both the funds are identical in CFs except the contribution and distributions for years 2023 and 2024 are interchanged. Specifically, *Fund B* returns a nominal $1 in 2023, which *Fund A* does in 2024. At the same time of these funds' existence, the stock market represented by the index in column 2 is volatile, experiencing both negative and positive returns annually.

The PME computations are performed as follows. Two portfolios named *Theoretical A* and *Theoretical B* are formed to mimic the CFs of the two funds. So in 2022, $75 is invested each in the two theoretical portfolios. In 2023, the $75 investment in each fund grows at 10% (the return on the index). *Fund A* calls for $50 worth capital, which is then added to *Theoretical A* portfolio which is worth $75 × 1.1 in 2023. *Fund B* distributes $1, which is then subtracted from *Theoretical B* portfolio ($75 × 1.1). These calculations are repeated in each row based on the cash flows of the fund and the index returns. Finally, in 2026, IRRs are computed for the two funds and their theoretical equivalents.[2]

2 - The cash stream for computing the theoretical portfolio's IRR includes the fund cash flows till the penultimate year and the final valuation of the theoretical portfolio.

The IRR spread for the two funds indicates that *Fund A* underperformed whereas *Fund B* outperformed. However, the real performance of the two funds is much more nuanced. Although *Funds A* and *B* have performed somewhat similarly, with even their IRRs indicating such similar performance, PME-based IRR spreads are very different. Why is this happening?

To the extent that private and public market valuations are correlated, as agreed by most market participants, both timing and company selection are important aspects of value creation in private markets. But by focusing too much on company selection, rather than timing, PME approaches reward managers more on their company selection rather than market timing. Given that GPs can manage their capital calls and distributions, this approach may lead to misleading conclusions.

For example, *Fund A* has called capital before a period preceding high growth in the market, i.e., 2024 whereas *Fund B* has returned capital prior to good market performance. This indicates that *Fund A* has timed the financial markets very well compared to *Fund B*. However, *Fund B* has been able to select superior companies to invest in as they have been able to return the same amount of capital as *Fund A* over the fund life, despite being poor on their timing. Although the IRRs and MOICs are very similar between the two funds, PME rewards *Fund B* more than *A*.

## 2.3 Why is it Important and How to Improve Valuation Methods?

Any improvement in valuation methods can have material consequences. To illustrate this, below is a demonstration of how flexible the range of valuations can be under different assumptions, followed by a discussion on why focusing on company-level valuations can improve the status quo.

### 2.3.1 Choice of Discount Rates

When performing valuations using the DCF approach, many GPs use fund target IRRs as discount rates. Gompers et al. (2016) find that the commonly marketed target IRRs in funds range from 15% to 35%, and it is often observable that GPs use marketed IRRs in their valuation. Below is an illustration of how the choice of a specific discount rate by a GP across its holdings in different sectors can affect its valuation of the company. For simplicity's sake, it is assumed that the CFs and growth rate are not controversial for the three considered companies, and DCF is used.

Company 1 is in a saturated sector with steady CFs and low future growth. Companies 2 and 3 are in growing sectors with relatively smaller CFs. Discounting the CFs and terminal value based on the growth rates presented in Table 5 at a 20% discount rate produces fairly similar valuations for the three companies ranging from $800 million to $850 million roughly. However, using a discount rate of 15% or 35% can produce widely different valuations. For example, at a rate of 15%, Company 3 is the most valuable, as growth is highly valued when discount rates are low. However, when discount rates are higher at 35%, then Company 1 is considerably more valuable as much of its value is in the form of near-term CFs.

Such a one-size-fits-all approach to valuation is flawed, as the CFs from a high-growth company are arguably riskier than those from a low-growth one. One dimension in which the riskiness of the CFs varies is across the industrial sector of the company. Thus, an improvement can be to consider sector-level discount rates that are also time-varying, such as those available for publicly listed industrial sectors (Damodaran, 2019). However, since these are for publicly listed stocks, an illiquidity premium needs to be considered for private companies. Based on a constant 5% illiquidity premium, the three companies have discount rates of 7.78%, 11.28%, and 14.12%, respectively, in January 2022. These inputs produce valuations of $1.89 billion, $1.08 billion, and $0.67 billion, respectively.

What is surprising in this illustration is that using such discount rates produces a valuation of Company 1 that is close to three times the valuation of Company 3. Only by addressing the constant nature of discount factors across sectors and time, one is able to get vastly different valuations. However, there is also scope to consider other risk factors apart from industrial activity, incorporate time-varying illiquidity risk premia, and also question the validity of CF streams, all of which can affect the valuation. Thus, DCF provides a framework but is too pliant to be of any use to be rigorously applied for private companies.

### 2.3.2 Focus on Company Level Valuations

The first step in improving the status quo is to go back to the basics and address the valuation at the company level. The current data available in private markets is usually based on the fund's reported NAVs that are aggregated and summarised across markets. However, this poses significant problems such as:

Table 5: Illustration of choice of discount rates

| Pvt. Company | Year ▶ Sector | 2022 | 2023 | 2024 | 2025 | 2026 | Growth |
|---|---|---|---|---|---|---|---|
| | | | Expected cash flows in $ millions | | | | |
| Company 1 | Retail (grocery & food) | 100 | 100 | 100 | 100 | 100 | 1% |
| Company 2 | Education | 80 | 80 | 80 | 80 | 80 | 5% |
| Company 3 | Hotel Gaming | 60 | 60 | 60 | 60 | 60 | 10% |

| Valuation in $ millions | | | | | | |
|---|---|---|---|---|---|---|
| Discount rate ▶ | 15% | 20% | 25% | 30% | 35% | Sector ERP + 5% |
| Company 1 | 1,056.6 | 830.6 | 689.8 | 591.8 | 519.1 | 1,891.3 |
| Company 2 | 1,108.2 | 799.2 | 565.1 | 530.8 | 457.6 | 1,079.6 |
| Company 3 | 1,521.1 | 839.4 | 601.4 | 476.1 | 397.2 | 667.3 |

1. Non-standardised valuation methods used within a GP's portfolio and across GPs, which when aggregated create distortions.
2. Ad-hoc and non-scientific choice of inputs and adjustments to valuation.
3. Combining realised CFs and estimated valuations as equivalent measures.
4. Same holdings in different GP portfolios can receive different valuations and at different frequencies for the above reasons, which creates further inconsistencies when valuation summaries are reported.

Thus, any improvement to valuation methods, should begin at the private company level and use standardised and objective approaches as proposed in this paper. Moreover, focusing on company level valuation has the following advantages:

- **Unit of account**: Private companies are the fundamental unit of accounts in private markets. For example, the size or leverage of an asset cannot be broken down any further than at the company level. Also, it does not make sense to pro-rate a company's sales or leverage to the holding proportion. Moreover, since the predominant investment strategy in private markets includes buyouts where the entire company is bought, it makes sense to focus on company-level valuation. Furthermore, the computation of firm-level valuation in a periodic manner facilitates the computation of all other metrics. That is from periodic company valuation, it becomes easier to compute unknown holding and fund level returns, but challenging to perform vice versa when company level valuation is unknown.
- **Variation in contracts**: The customised nature of contracting between LPs and GPs can accommodate various arrangements of reporting, fees, and performance allocation. For example, it is fairly common for GPs to offer preferential treatment to their LPs. Often the justification for preferential treatment relies on the size of LP investment and how early the investment was made in the fund.[3] However, information asymmetries between GPs and LPs allow for performance reported to one LP to vary from that reported to another, thus raising validity questions on fund-level net-of-fees metrics for each LP.
- **Lack of repeat transactions**: Even when focusing on company level metrics, another question remains as to whether to model returns or valuations. Factor models for publicly listed stocks usually model log returns as the dependent variable but, due to the sparseness of transactions in private markets, returns are usually not computable. In other words, a private company rarely transacts multiple times even over decades, making return computations impossible.

3 - Even the regulatory changes brought to the private markets by the Securities Exchange Commission (SEC) in 2023 do not forbid preferential treatment and only require disclosures to all LPs on such arrangements.

Thus, valuation metrics are preferable as the modelled variable.

## 2.4 What Can a Better Approach do?

In this section, the benefits one can get from using a better approach to private company valuation, and more specifically the benefits from using the proposed approach are presented.

### 2.4.1 Advantages of the Proposed Approach

Our proposed approach to the valuation of private companies is novel and significantly better than other alternatives for the following reasons:

1. **Transforms weak signal into information**: The proposed model can use weak and biased transaction data and transform it into reliable and useful information through the dynamic linear model, which can learn the unbiased factor prices through time.
2. **No subjectivity problem:** The proposed method relies on building a formal factor model exclusively with transaction data from private markets, thus eliminating any subjective component in valuation, such as adjustments needed to multiples for illiquidity, size, leverage, etc.
3. **Control for observables:** An explicit factor model approach based on characteristics enables observable company characteristics to be controlled in a continuous, objective, and clear manner, i.e., no implicit adjustments are required to account for differences from a comparable company. This supports the computation of how much-changing characteristics can affect valuation and attribute changes in valuation to characteristics. For example, questions such as: "If revenue increased by 10%, how much can valuation improve?" or "How much of the valuation increase is due to expansion in the profitability of 3% points?" become trivial to answer with a factor model.
4. **In-house classification scheme:** Our approach creates a taxonomy of private companies that captures various segments of private asset markets along multiple dimensions such as their industrial activity, phase of growth, position in the value chain, output characteristics, and type of revenue model. It also helps to capture several risk factors that affect valuation. The in-house taxonomy is called PECCS™ or *PrivatE Company Classification Standard*. Details of the taxonomy and its approach to segment individual private companies into these segments are available in EDHECinfra and Private Assets Documentation (2023).

Current vendors and practitioners often rely on existing public industry classification schemes (e.g., MSCI's GICS) to segment private companies. Such classification schemes are focused on public markets and are not comprehensive enough for private companies where the lack of historical returns confounds understanding of key risk factors, as explained below.

For listed stock, risks can be parsimoniously inferred from estimating their $\beta$ by regressing its historical return on different risk factors that affect valuation. For example, the growth prospects of an asset are expected to affect valuation (and hence returns). In the case of public equities, this can be parsimoniously expressed through the $\beta$ of the stock to the Fama-French value factor (Fama and French, 1993). The value factor is constructed as the return of a long-short portfolio that is long (short) on stocks with the lowest (highest) book-to-market ratios.

However, in the case of private companies with data paucity, even if such a factor can be constructed based on past observable transactions, asset-specific $\beta$s are unobservable. An equivalent way to capture such an exposure to growth factor can be to segment companies, based on some criteria, into categories of growth, for example, startup, growth, and mature, and observe whether valuations systematically differ across these categories.

Thus, the PECCS™ classification scheme benefits the valuation approach. Specifically, the multiple dimensions of the classification scheme, when calibrated properly on past transactions of other private companies, help arrive at a quantitative premium or discount for the various characteristics of private companies, thereby compensating partially for the lack of observable price histories.

5. **Overcomes sparseness problem:** Another concern with the comparable multiples approach is the 'sparseness' problem, where one cannot observe a similar and/or recent transaction for the asset being considered. The sparseness problem is overcome in our proposed approach by relying on dynamic linear models that allow for time-varying factor prices, thus making use of more transactions from a longer historical period.
6. **Robustness:** Our models rely on a large and representative sample of transactions from 1999 till 2022, thus allowing them to generate robust estimates of factor prices. Also, by using a long time period of transactions and estimating a dynamic linear model, time-varying factor prices are accommodated (i.e., changing investor appetites through time). Finally, relying on a large sample of transactions also means that noisy and biased inputs (i.e., transaction prices with idiosyncratic features) can be utilised to obtain unbiased valuation estimates.
7. **Granularity:** Using transaction data on private companies, the factor model, when applied to financial and other company-related information, can produce valuation estimates at the asset level and over time. Any other alternative valuation data in the market, even when available at the asset level, relies solely on GP-reported valuations and hence suffers from all the associated biases, such as return smoothing, staleness, and the absence of return volatility.
8. **Precision:** Our proposed models can produce very precise estimates of valuation by leveraging all the historical transaction information. Relying on contemporary developments in finance academic research, in future work, it is also possible to estimate bid-asks for the valuation estimate, representing the good deal bounds of Cochrane and Saa-Requejo (2000). That is, a point estimate from the factor model can be supplemented with a range of possible valuations that can be considered to be good deals. Moreover, the accuracy of segment-level estimates is higher than the asset level, as by design the errors in predictions cancel out each other, thus providing far more meaningful benchmarking alternatives for portfolio allocation and monitoring.

Thus, a model-based approach calibrated with transaction data and novel risk factors can transform sparse, biased, and noisy data into good information that can help private markets.

# 3. Literature Review & Potential Factors

Numerous studies evaluate PE returns, the predominant channel to invest in private companies, based on contributed and appraised NAV data from GPs. For example, Kaplan and Schoar (2005) find that PE gross returns outperform the S&P500 while net returns are on par with the S&P 500. Also, there is strong persistence in performance by GPs. Ilmanen et al. (2019) argue that a leveraged small-cap equity index is a better benchmark for PE performance than broad unleveraged indices and, based on such benchmarks, net-of-fees performance has been decreasing over time.

However, few studies investigate the valuation of the underlying private companies and the factors that affect their returns, largely due to the paucity of regular transaction-based prices of private companies, frequent fund-level cash flows, and importantly a lack of thorough fundamental financial data for private companies.

In this chapter, we provide a brief discussion of broad transaction prices based models, followed by a discussion of potential factors that can explain transaction prices.

## 3.1 Transaction Based Pricing Models

Transaction-based pricing models can overcome the flaws in common approaches to valuation as discussed in Chapter 2. For example, transaction-based methods based on established statistical approaches can help overcome the qualitative and subjective natures of DCF and multiples-based approaches. Moreover, by using transaction data, both the smoothing and lagged nature of appraised values can be overcome. Simple transaction based pricing models can focus on a sample of repeat sales that eliminates the role of characteristics in prices, while more advanced hedonic models can take into account asset characteristics in a more formal manner.

Repeat sales indices, popular in real estate literature, can estimate the percentage return on assets by focusing on those that have been sold at least twice. As early as Bailey et al. (1963), such approaches have been used to estimate price indices for real properties. By regressing the return on specific assets based on transaction prices observed at least at two points of time, on time indicators, the average returns can be estimated. The repeat sales method can eliminate the role of characteristics and quality on prices, and allow easy computation of average indices. However, in the case of private companies, the characteristics (e.g., size, profitability, etc) are more dynamic than those of real properties and many assets may not transact twice over long periods of time. Hence such an approach would fail to capture the representative valuation and the role of changes in characteristics.

Hedonic pricing models are extensively used in the real estate literature in pricing housing stock or commercial property (Malpezzi et al., 2003). Such models identify both internal characteristics and external factors that can affect the price of an asset, and when calibrated using transactions, can provide estimates of prices. These models are usually estimated in two stages where the first stage estimates a reduced form regression of transaction prices on characteristics of the asset. Under the assumption that the supply of characteristics is perfectly elastic, a second stage demand estimation is performed to infer the prices of characteristics (e.g., Clapp and

Giaccotto, 1992). The purpose of a second stage is to go beyond the estimation of a hedonic price surface and recover the structural demand parameters for individual asset characteristics.

As an example, Blanc-Brude and Tran (2019) build a hedonic model to explain the prices of unlisted infrastructure based on their transaction prices and the characteristics of the asset. The estimation is performed using a dynamic linear model that allows for time-variation in factor prices. It is possible to follow a similar approach to estimate the factor prices, based on observed transactions in private companies. To implement, a set of factors is selected that can sufficiently and parsimoniously explain prices. The dynamic estimation allows for evolving coefficients which correct for the biasedness arising from infrequent observations and are also able to filter out the noise from each individual transaction. In Chapter 5, this approach is described in more detail.

For this approach to perform well, it is necessary to identify a set of potential factors that can adequately capture variation in valuation. Next, some of the key systematic factors that can explain valuation and the economic intuition behind these factors' association with valuation must be reviewed. Additionally, some private equity-specific institutional details that could potentially be associated with private company valuation should also be explored.

## 3.2 Potential Factors

### 3.2.1 Size

Since Fama and French (1992)'s work, size has been a reliable factor in pricing securities of various asset classes including stocks, bonds (Fama and French, 1993), infrastructure assets (e.g., Blanc-Brude and Tran, 2019), etc. Prior work has attempted to contextualise the outperformance of investments in small firms in terms of distress risk (Fama and French, 1993), credit risk (Vassalou and Xing, 2004), illiquidity (Amihud and Mendelson, 1986), tax-loss selling (Rozeff and Kinney Jr, 1976), etc. Small cap premiums are more important for private companies which, when compared with public companies, are smaller, less frequently traded, and lack proper financing facilities, thus making them more risky. Ceteris paribus, smaller private companies are expected to have lower valuations given that they are riskier and require higher returns. Moreover, private equity performance also reveals that the internal rate of returns (IRRs) of small-cap buyout funds are consistently higher than those of buyout funds focusing on mid- and large-cap stocks, even when examined across performance quartiles (McKinsey, 2023).

However, size is also inversely proportional to information uncertainty, making smaller assets more prone to mispricing. Further, this phenomenon is likely exacerbated in private capital markets due to their illiquid and lumpy nature. Thus, size can also have a positive effect on valuation, and hence empirical analysis is resorted to uncover the exact relationship.

### 3.2.2 Leverage

Theory predicts a direct link between capital structure and required returns, with highly leveraged firms being riskier and thus requiring higher returns. However, empirical evidence is mixed, ranging from positive to nil to negative association between leverage and required returns (e.g. Gomes and Schmid, 2010; George and Hwang, 2010), with several alternative explanations. For example, firms may endogenously choose lower leverage when they have high required returns. Similarly, mature and safe firms might choose a high level of indebtedness. In addition, since financing and investment opportunities are highly correlated, leverage choices can indicate growth options and be reflected in required returns. Thus, leverage is likely to have a complex relationship with required returns, and hence valuations.[1]

Furthermore, in the context of private companies, a PE investment is usually an opportunity to recapitalise the firm. Along with operational and governance engineering, financial engineering is a key pillar of value creation for PE funds (e.g. Gompers et al., 2016). For example, Demiroglu and James (2010) document that among a sample of leveraged buyout deals of publicly listed companies, the average equity portion of the deal's financing is about 35%, which decreases further when credit standards are lax. The rest of the deal is usually financed through a combination of unsecured loans, first-lien bank debt, and secured bank revolvers. Such leverage choices during the deal indicate the acquirer's expectation of returns. In a traditional sense, this could indicate a higher required return rate. However, as argued above, the marginal leverage choices may also correlate with the target's maturity and investment opportunities, hence confounding the prediction of leverage on valuation.[2]

### 3.2.3 Growth

Fama and French (1992) demonstrate that value stocks (i.e., stocks with higher book values relative to their market values) earn higher returns. Conversely, growth stocks (i.e., stocks with lower book values relative to market values) earn lower returns. Rational explanations for such phenomenon suggest value stocks being more risky, such as during bad times when value firms cannot adjust investment with the business cycle (e.g. Petkova and Zhang, 2005). Market inefficiency explanations indicate that naive investors may make systematic errors about risk and earnings (Lakonishok et al., 1994). Naive investors may fail to distinguish between systematic and idiosyncratic components of risk. Idiosyncratic risk components can be diversified away and do not require return premiums. Similarly, naive investors can also make systematic errors about the earnings of value stocks as they become excessively pessimistic when earnings are poor.

Both arguments are also relevant when it comes to private companies, which are more sensitive to business cycles (e.g., Crouzet and Mehrotra, 2020) and, as argued above, trade in highly illiquid and lumpy markets that are more prone to mispricing. Thus, it is anticipated that high-growth private companies experience higher valuations.

### 3.2.4 Profitability

Novy-Marx (2013) finds that profitable firms enjoy higher returns than less profitable firms while ironically at the same time enjoying higher valuation ratios, posing a puzzle. A proposed explanation is that while productive assets are deemed valuable, investors also require higher returns as these high profits are the equivalent of having leveraged claims on revenue. Hou et al. (2015) suggest that if the investments of a firm are considered as fixed, higher profitability indicates higher costs of capital because if the cost of capital were lower, then firms would invest and profit more.

Novy-Marx (2013) also shows that adding profitability on top of a value strategy in a portfolio reduces the strategy's overall volatility despite doubling its risk exposures. Since more productive firms have a growth tilt (i.e., low book-to-market ratios), profitable firms provide a good hedge to an investor's value exposure. Given such effects of profitability on returns in public stocks, it

1 - Expected or required returns and valuation are inversely correlated, and hence a factor that predicts a higher expected return, indicates a negative effect on valuation. Due to the unobservability of returns of private companies, caution is advised when interpreting factors that predict returns of stocks to how they affect private company valuation.

2 - Thus, in the context of private company transactions, two types of leverage are considered, including the existing leverage and the additional leverage brought through the transaction. Additionally, leverage can also be related in a nonlinear manner with valuation, and hence such possibilities are explored by experimenting with square of leverage as potential factors.

is also expected that profitability is a key factor in the valuation of private companies.

### 3.2.5 Market Characteristics

Transactions in private companies do not happen in a vacuum and rely on prevailing market conditions. The demand for a private firm's products and services and supply of capital are closely linked to overall economic health. For example, Gompers et al. (2008) find that the volatility in the VC markets is very closely tied to that of the public markets, which they argue is a rational response of fund managers in allocating capital to attractive investment opportunities as signalled by the public markets.

In addition, interest rates and their term structure impact the valuation of private companies through their effect on discount rates and investors' maturity preferences. The spectacular growth in PE coincides with a long period of benign interest rates such as the first two decades in the twenty first century, when GPs can arbitrage the low yields on debt with high yielding private companies (e.g., Blundell-Wignall, 2007).

Market conditions also affect deal clustering dynamics, i.e., the concentration in the distribution of transactions from a specific sector or geographic region over time. For example, Buchner et al. (2020) find that buyout funds make more correlated investment choices, i.e., herd more when market conditions are adverse and competition for capital is higher.

Also, institutional conditions of private capital markets such as investments entering PE funds, the amount of dry powder, i.e., committed but uncalled capital, and size preferences of private market investors can affect transaction prices. As well, characteristics reflected in public equities markets such as its liquidity, volatility, investor preferences for growth, and macroeconomic conditions (e.g., expected GDP growth) can affect preferences for private companies, and hence their valuation.

In addition to the above concepts that are expected to be the core determinants of a private company valuation, several additional factors that may be related to valuation are also considered, as described briefly below.

### 3.2.6 Age

Younger firms, whether private or public, have fewer track records and are subject to more information uncertainty. Prior work has shown that listed firms with high uncertainty, proxied by their age, earn lower returns. The reason for such underperformance is that uncertain firms attract overconfident traders, thereby limiting rational arbitrage (e.g. Jiang et al., 2005). Even among PE, an increase in capital supply leads to tougher competition for deal flow (e.g. Ljungqvist and Richardson, 2003), which can lead to mispricing of younger assets. Thus, a negative relationship between the age of private firms and valuation is anticipated.

### 3.2.7 Human Capital

A key source of value creation in private companies through operational engineering is through the effective management of human capital. When nurtured and deployed properly, human capital can lead to sustained competitive advantage (e.g. Hall, 1993), especially in private firms that are yet to realise their full potential. However, larger employee bases increase coordination costs in activities such as teamwork (e.g. Onal Vural et al., 2013) and hence can lead to lower valuation of firms. Moreover, larger employee bases reduce the effectiveness of value creation through operational engineering due to increased complexities.

In contrast, a company with a large employee base provides additional opportunities for PE firms to restructure the portfolio company to create value, such as through reductions in the workforce and renegotiation of wages and pension schemes, etc. Thus, ceteris paribus, the relative size of employee bases of private companies provides both opportunities and threats for value creation, making it an important factor in the valuation.

### 3.2.8 Technology

Economists have long argued that technological innovation is a key driver of economic growth. Private companies significantly contribute to innovation and facilitate the creative destruction and reallocation of capital to innovation. Prior work in innovation suggests that publicly listed companies face short-term pressures that make it difficult to invest in innovation (e.g. Fang et al., 2014). Moreover, studies also find that PE ownership does not reduce innovation but rather focuses it and enhances its commercial potential (e.g. Lerner et al., 2011). Thus, innovation can be a key differentiator of value, especially among private companies, and thus is an integral part of the factor model for private company valuation.

### 3.2.9 Industry Concentration

Hou and Robinson (2006) find that firms in concentrated industries earn lower returns, i.e., have higher valuations. High barriers to entry reduce the riskiness of companies in concentrated industries. In addition, such companies can under-invest in innovation as they lack the incentives in the absence of competition and thus require lower returns. A firm's operating decisions arise from strategic interactions in the product markets, and thus the riskiness of its cash flows is closely linked to the industry structure. These arguments suggest that industry concentration positively affects their valuation. Moreover, the market share of the focal company being valued can capture the sensitivity of the specific company to its industry concentration. For example, a large company in a concentrated industry may face lower risk and hence deserve a commensurate valuation for its level of risk, while for a small company the odds are stacked against it even in a concentrated industry.

### 3.2.10 Transaction Characteristics

Private equity GPs are increasingly engaging in buy-and-build strategies where a portfolio company acquires a smaller target private company, often referred to as an add-on transaction in private capital markets. Such add-on transactions can be associated with valuation as they hint at future business strategy, anticipated synergies that the target company has with the portfolio company, and also exit options for the target company, thus making this an important consideration in the transaction amount paid.

Additionally, whether the target company is a publicly listed company can have an effect on its valuation. For example, a publicly listed asset can be taken private at a systematically higher valuation due to the absence of private company discount (e.g., Koeplin et al., 2000) and the lower level of information asymmetry between the buyer and the seller. The information environment surrounding a publicly listed company is usually superior to that of private companies on account of increased disclosures as well as continuous scrutiny of numerous institutional investors.

Also, the existing owners can have implications for the valuation. Increasing investment in PE funds and their short lives, despite the longer time companies stay private (e.g., Ewens and Farre-Mensa, 2020) creates a timing mismatch. This has led to increased transactions in the GP led secondaries markets. Such secondary transactions raise concerns about capital deployment (Degeorge et al., 2016), liquidity, performance, seasoning, and efficiency gains (e.g., Wang, 2012). Hence, such deal characteristics have the potential to affect valuation.

Furthermore, in buyout transactions, existing investors might choose to stay invested (i.e., roll along or retain their stake). They may be a PE investor from before rolling along their stake or a founder who wishes to retain their stake for many reasons. Such transactions may be characterised by less than 100% buyouts, a more incentive-aligned management, lower cash outlays, and hence lower leverage, or even be linked with higher coordination problems, creating implications for the valuation.

The above factors proposed were also posed to private equity fund managers in a survey by *EDHECInfra and Private Assets Research Institute*, who were asked to select the factors they considered as important, rank the factors in terms of their importance, and also indicate whether they expected a positive or negative effect on valuation. These results based on 95 responses are presented in Figure 4. The responses are in line with expectations, with the top three important factors having a positive effect on valuation (in terms of the number of respondents) being **Growth**, **Profitability**, and **Revenue**. The size of the bubbles in the figure indicates the number of respondents. In terms of rank, these three factors are also perceived as being more important.

### 3.2.11 PECCS™

As described earlier, several characteristics of public assets – which are usually taken for granted and computed as $\beta$s of return factors – need to be carefully defined, segmented, and measured for private companies owing to the deficiency in historical valuation. For this purpose, PECCS™ or *PrivatE Company Classification Standard*, a taxonomy of private companies, has been created by the *EDHECInfra and Private Assets Research Institute*. It captures various segments along multiple dimensions that can affect valuation risks, details of which are published in EDHECinfra and Private Assets Documentation (2023). The key pillars of private companies (i.e., dimensions) which are expected to adequately measure the systematic risk exposures include its 1) industrial activity, 2) type of revenue model, 3) phase of growth, 4) customer model, and 5) value chain characteristics.

Each pillar in PECCS™ is independent, i.e., captures a dimension of the private company not included in another. In addition, within each pillar, the categories or classes are exhaustive, i.e., they capture all potential possibilities. Also, in PECCS™, it is ensured that a company can only belong to a single category in each pillar, i.e., classes are mutually exclusive, unless the company can be divided into separate units, in which case, each of the units can only belong to a single category in each pillar.

Table 6 presents the classes in each of the PECCS™ pillars. The detailed descriptions and classification methodology are included as part of EDHECinfra and Private Assets Documentation (2023).

Apart from these dimensions, geography and technology could also have valuation implications. In order to keep the model sparse, these dimensions are captured through other variables in the model instead of introducing additional formal pillars.[3] For example, measures of interest rates, term spreads, and macroeconomic variables can absorb variation in valuation due to geography, and measures such as count of patents, product uniqueness, etc., can absorb the effect of technology on valuation.

3 - Note that the final model includes indicator variables for each category under every PECCS™ pillar, which can then summarise the relationship between the category in the pillar and valuation. Thus, determining the appropriate pillars and classes within each segment requires managing trade-offs between adopting numerous classes that capture all the variation and a lack of a decent sample size to estimate the classes' effect on valuation in a robust manner.

Figure 4: Key factors in private company valuation according to survey respondents

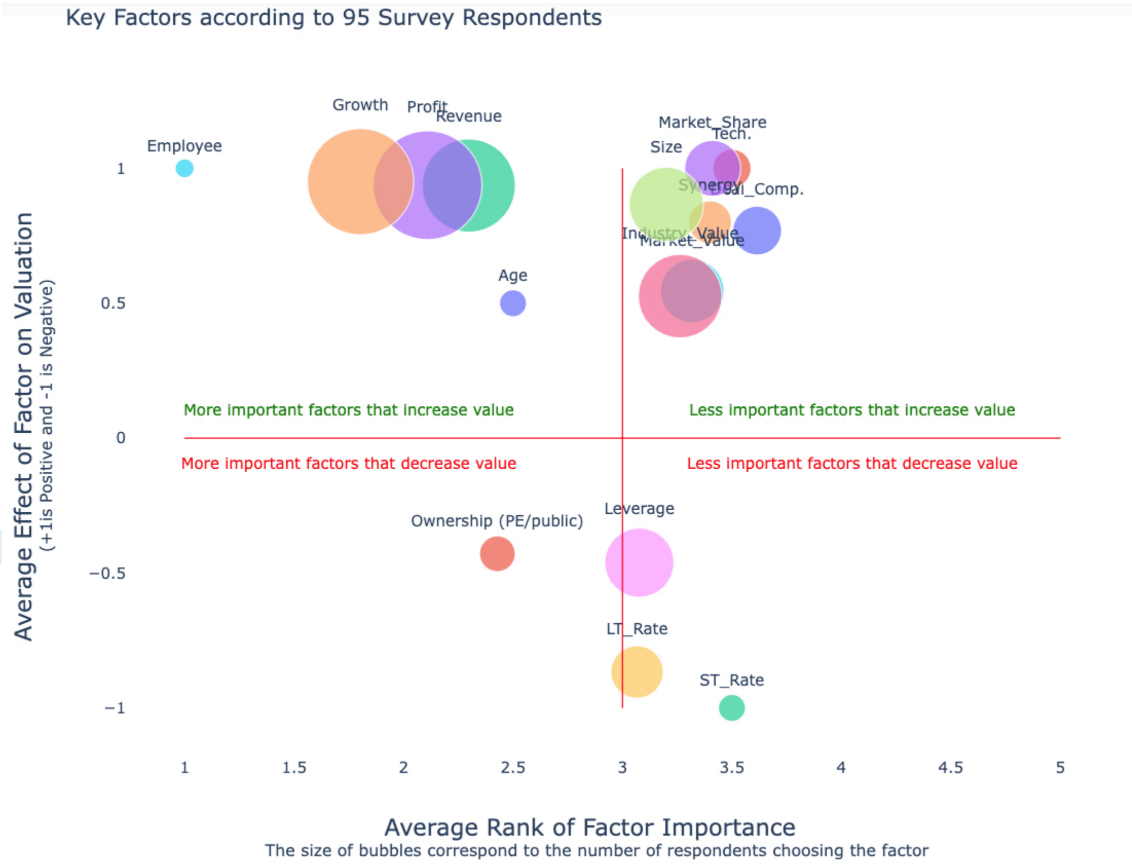



Table 6: PECCS™ pillars & classes

| PECCS™ segmentation of private companies | | | |
|---|---|---|---|
| PECCS™ Pillar → | Activity | Revenue Model | Lifecycle Phase |
| Description → | Types of industrial activities that sufficiently delineate private investment opportunity set | Key principles that underlie revenue generation, with implications for the sustainability, scalability, and predictability of the revenue | Stage of business development with implications for strategy, growth, operations, and investments |
| Classes within PECCS™ Pillar | | | |
| PECCS™ classes → | Education & public<br>Finance<br>Health<br>Hospitality & entertainment<br>Information & communication<br>Manufacturing<br>Natural resources<br>Professional & other services<br>Real estate & construction<br>Retail<br>Transportation<br>Utilities | Advertising<br>Production<br>Reselling<br>Subscription | Startup<br>Growth<br>Mature |
| PECCS™ segmentation of private companies | | | |
| PECCS™ Pillar → | Customer Model | Value Chain | |
| Description → | Output characteristics and the management distribution prefer-ences among selling directly to individual consumers and/or businesses, with implications for business strategy, marketing, customer engagement, and customer loyalty | Tangibility of output with implications for supply chain design, working capital, inven-tories, after-sales service, and supply chain information management | |
| Classes within PECCS™ Pillar | | | |
| PECCS™ classes → | Business focused<br>Consumer focused | Products<br>Services<br>Hybrid | |

Summarising the above discussion, Table 7 presents the list of factors explored along with the motivation for the factors (e.g., from academic work or others), and potential proxies that can measure the factor.

In conclusion, the above factors are expected to affect private company valuation. Many of these factors are motivated by asset pricing literature concerning stocks and bonds and hence are well grounded in theory. Although these measures represent a rather exhaustive list of factors, it is possible for more latent measures that may affect valuation, but which in the case of private companies may be unobservable, even in a large sample of transactions.[4] Additionally, a parsimonious representation of the multifactor model is pursued, thus preferring a simpler model that can be a better fit for the purpose of private company valuation.

4 - See for e.g., Green et al. (2017) for the proliferation in return predicting factors and how they often do not add much independent information.

Table 7: Potential factors and sources

| Factor | Source | Potential proxies |
|---|---|---|
| Private Company Characteristics | | |
| Size | Prior research & survey | Total assets, revenue, enterprise value |
| Growth | Prior research & survey | Sales growth, book-to-market ratio, Age |
| Profitability | Prior research & survey | EBITDA margin, EBIT margin |
| Leverage | Prior research & survey | Book leverage ratio |
| Human capital | Prior research | Ratio of employees to revenue, employee growth rate, productivity measures |
| Technology | Prior research & survey | Number of patents, indicator for hitech companies, similarity with rivals in terms of business description |
| Industry concentration | Prior research & survey | Herdinfahl Hirschman Index of industry sales, market share of industry |
| Transaction Characteristics | | |
| Financing | Private market features | Transaction leverage |
| Ownership | Private market features & survey | PE backed, control, public company |
| Strategy | Private market features & survey | Add on |
| Market Characteristics | | |
| Public markets | Prior research & survey | Public market valuation, sectoral valuation, volatility, liquidity, return factors in public markets |
| Private markets | Prior research & private market features | Transaction herding, dry powder, size factor in private markets |
| Macro characteristics | Prior research, private market features, & survey | Interest rates, term spread, Forex change, GDP growth, Inflation, Emerging market transaction |
| Size private factor | Prior research & private market features | Valuation transaction premium of small cap over large cap private companies |
| PECCS™Pillars | | |
| PECCS™ classes | Prior research, private market features, & survey | Indicators for every class in each pillar |

# 4. Data

This chapter provides the details of the sample construction and summary statistics of the sample. The key challenge in estimating the returns of private companies is the unavailability of regularly observed prices. For example, over the past 20 years, the average private company (for which an investment was observed at least once from PE) transacted on average just 1.15 times (excluding public listings as exits), indicating that regular return metrics cannot be computed in the traditional manner of listed equities. This lack of liquidity also means that any data on transaction prices has only information on biased factors, thus complicating the estimation of factor prices.

## 4.1 Sample Construction

We obtain a list of PE investments in private companies using data on global private firm deals and financials obtained from from *PitchBook*™, a *Morningstar* company.[1] To observe a decent number of observations per year, the sample begins in 1999 as transactions are few and far in between during earlier years.

For modelling, the **P/S** ratio is preferred over other ratios such as **P/EBITDA**, as EBITDA when negative renders **P/EBITDA** meaningless. Also, private companies can make several adjustments to EBITDA, making it less standardised by way of comparison. Focusing on valuation rather than performance is for mechanical reasons. The lack of repeat transactions in private companies hinders the computation of returns. Using valuations allows the factor model to maximise available data. For example, if only returns were used in the factor model, the majority of any sample would need to be discarded as multiple companies do not transact twice even over long periods of time.

Using PitchBook, we consider transactions that have a minimum size of $10 million, have been completed successfully, are regarded as PE investments (with the exception of PIPEs or Private Investment in Public Equities), and involve a private company with most recent sales greater than $5 million. Additionally, we exclude transactions with key deal information or the most recent financial information that is missing. Transactions where the **P/S** or Price/Sales ratio falls in the bottom or top five percentiles are also excluded. Dropping these outliers enables the model to better capture the average transaction.

These steps yield a global sample of 5,438 transactions during the 1999-2022 period. To illustrate the sample construction steps, Table 8 reports how each of the key filtering criteria affects the sample, beginning with more than 3.5 million observations in PitchBook. The percentages in each row indicate the reduction in sample size compared to the previous row. The percentage reduction is reported instead of absolute numbers as PitchBook and other similar vendors update transactions in real time, making the absolute numbers specific to the time. Thus, percentages can provide a more meaningful snapshot of arriving at the sample. For the sake of brevity, certain variable constructions that result in a loss of less than 10 observations are not recorded in Table 8.

1 - Brown et al. (2015) find that during 1984-2010, among data providers that report private equity performance, including Burgiss, Cambridge Associates, PitchBook, and Preqin, PitchBook has strong performance in terms of funds coverage and capital committed, especially in North America. This is ideal for the empirical exercise to model the valuation of private companies transacted by private equity funds. Moreover, detailed transaction data is available in PitchBook, while not disclosed by Burgiss or Cambridge Associates, due to confidentiality clauses in their contributed data sourced from GPs.

Table 8: Summary of filtering transactions

| Step | Description | % reduction |
|---|---|---|
| | Filtering within PitchBook™ | |
| 1 | Transaction Types: PE excluding PIPE | -95.00% |
| 2 | Status is completed | -1.32% |
| 3 | Deal size of $10 million | -81.72% |
| 4 | Revenue of $5 million | -66.36% |
| | Filtering during processing | |
| 5 | Missing revenue | -20.56% |
| 6 | Missing founding year | -6.97% |
| 7 | Missing profitability | -30.84% |
| 8 | Missing public market data | -0.08% |
| 9 | Missing employees | 4.94% |
| 10 | Exclude Outliers | -10.68% |
| | Final sample of 5,438 transactions | |

### 4.1.1 Sample Distribution

Figure 5 shows the distribution of the number and size of transactions as a proportion of the final sample. Based on counts, the UK and the US account for approximately 26.7% and 25.2% of the sample number of transactions, respectively. The remaining countries in Europe and Central Asia, when considered as a group, constitute 31.3% of the sample. Thus, the distribution of the sample is globally diversified, which increases the generalisability of the findings. Moreover, the sample is also consistent with the distribution of private equity funds (e.g. Wallach, 2020). The second chart in Figure 5 shows the same distribution, considering the aggregate size of transactions. A slightly different picture emerges, with the US accounting for more than 51.3% of the sample by size.

Next, transactions are broken down by size and industrial activity, shown in Figure 6. The majority of the sample falls in the $1.5 to $5 billion range at 30% while the mega deals (> $10 billion deals) constitute 19% of the sample. The segmentation by industrial activity produces a somewhat uniform segmentation of the sample transactions. Much like public markets, *Information and communication* is the largest industrial activity in the sample, accounting for 26% of deals, followed by *Manufacturing* at 22%. Notably, *Financials* and *Health* have smaller weights than in public indices. For example, Finance and Healthcare constitute 14% each in the S&P 500 in March 2023.[2]

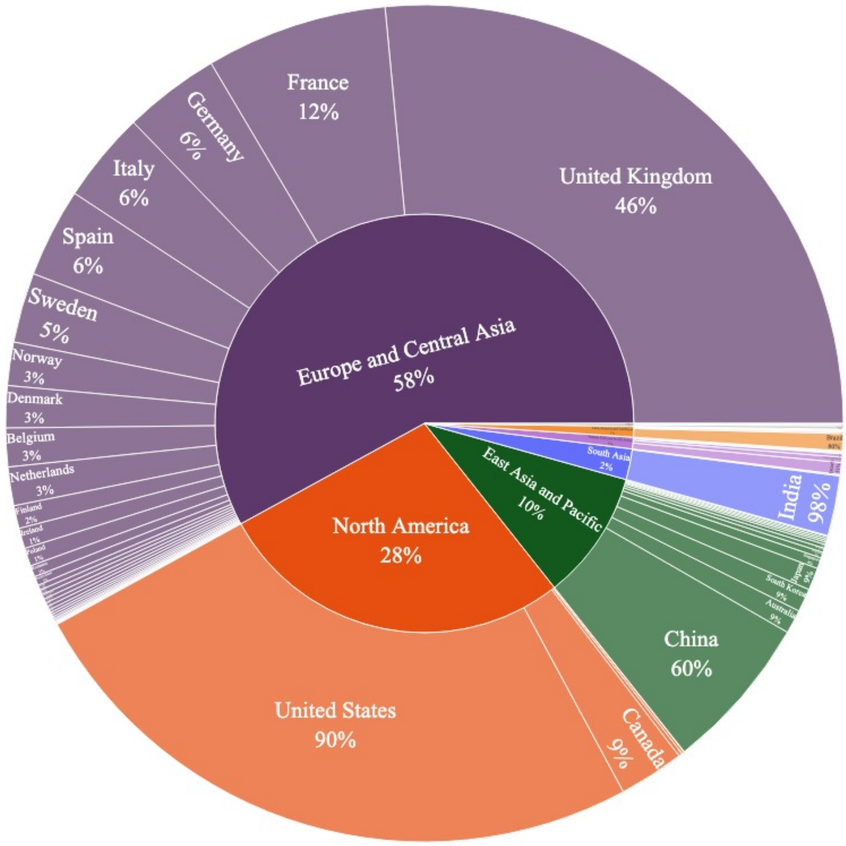
To obtain a clear picture of how these industrial activities have fared over the years, Figure 7 presents the number and size of transactions per year during the sample period of 1999 to 2022. *Manufacturing* and *information and communication* constitute the two largest activities in the sample in terms of number of deals. In terms of transaction size, *information and communication* form a larger proportion of transaction value indicating that this activity experiences larger transactions on average.

Having established that the sample is large, representative, and diversified in terms of geography, time period, and industrial activity, we next explore the choice of an appropriate valuation metric for private companies. The chosen metric needs to be comparable across companies in different sectors and countries. Standard return metrics are not useful, as the median private company is not traded

2 - https://finance.yahoo.com/quote/SPY/holdings/ - retrieved on March 20, 2023.

Figure 5: Sample distribution across countries by number and size of transactions

Based on # of Transactions




Based on Size of Transactions

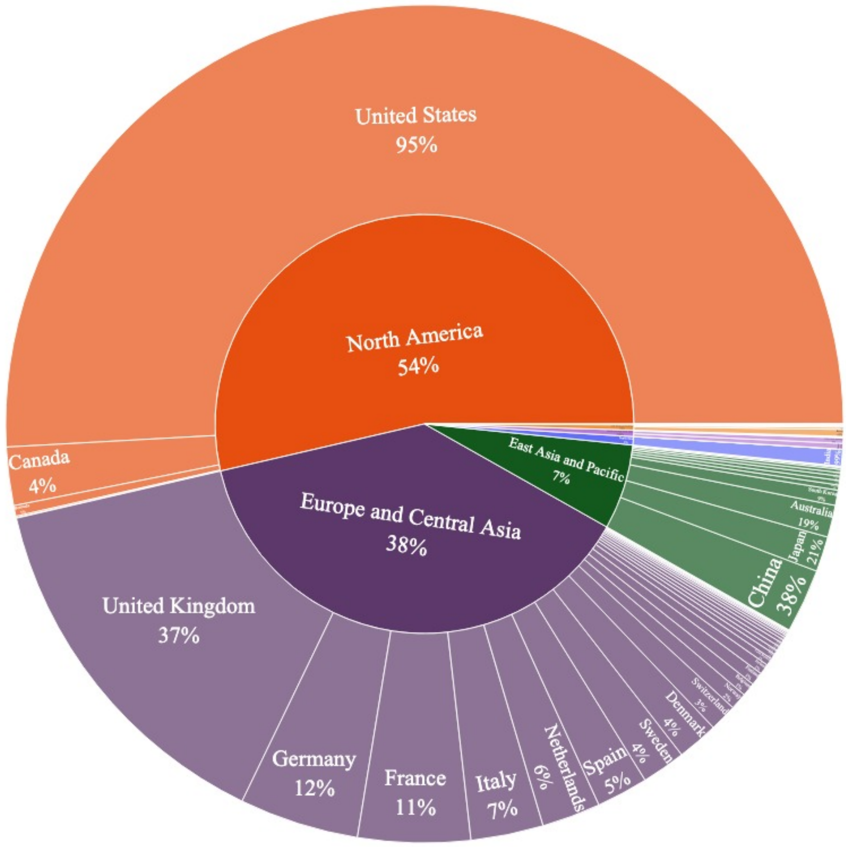
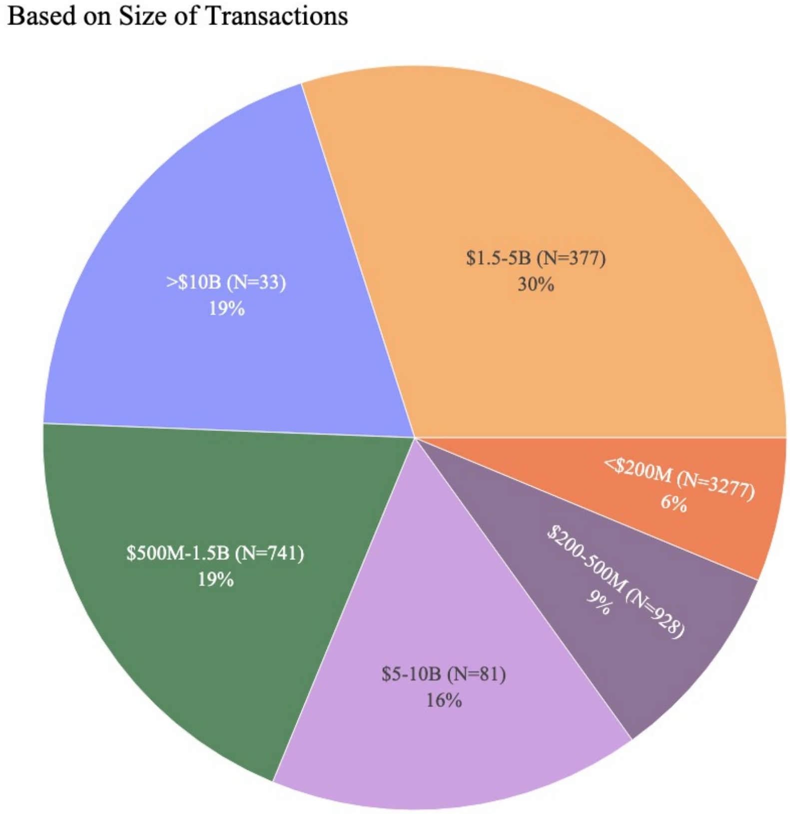


Figure 6: Sample distribution by size and industrial activity



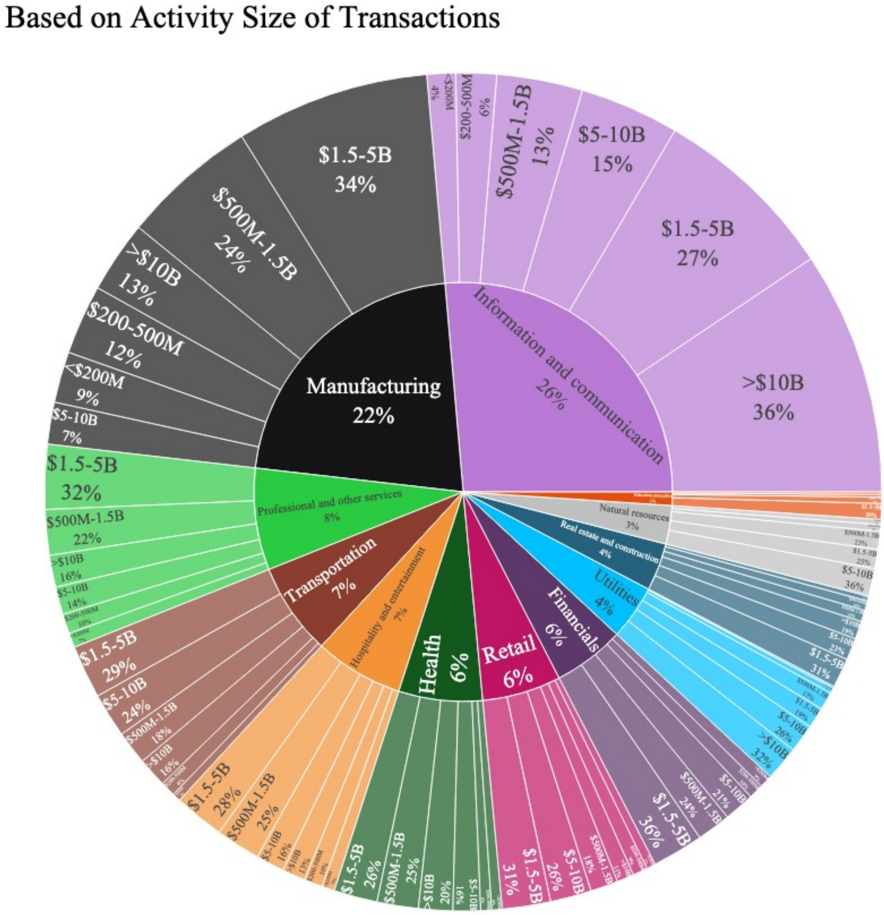
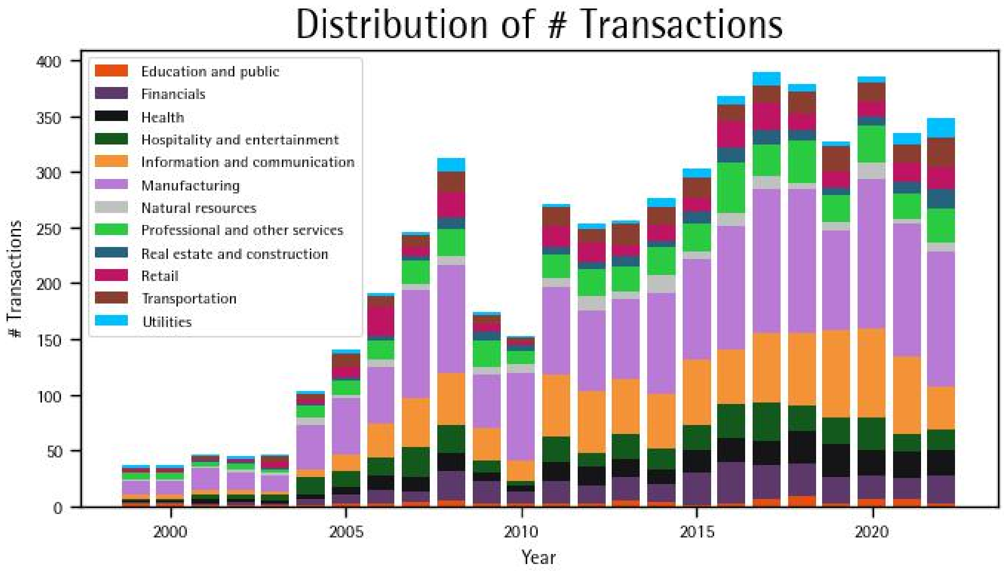
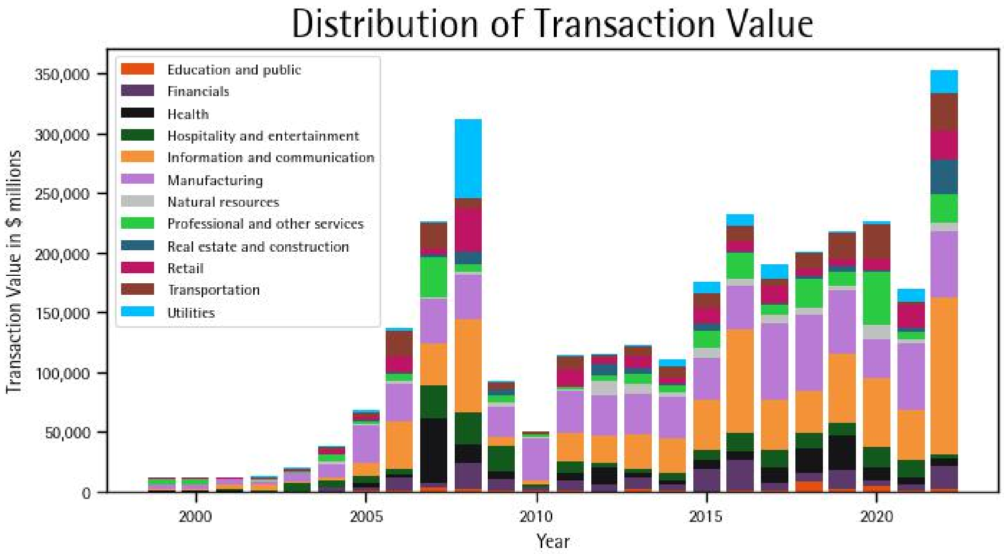



Figure 7: Sample distribution across the years




more than once in the sample. Internal Rates of Returns or IRRs are also not suitable, as discussed in Section 1.1.1, and because they are not systematically reported. Furthermore, even if return metrics are computable, the time period of measurements is far from regular.

Thus, price based valuation measures are appropriate, where the price is measured as the valuation of 100% equity based on the transaction price recorded in *PitchBook*$^{TM}$. Note that the observed transaction can potentially be for less than 100% of equity value. In fact, the sample median of equity percentage purchased in the transaction is 79.5%. Next, to standardise price across sectors, time, and currencies, it is measured as the ratio over sales in the previous financial year, i.e., P/S ratio.

Standardising by sales produces more statistically desirable measures than earnings or EBITDA (earnings before interest, taxes, depreciation, and amortisation). More to the point, earnings or EBITDA can often be negative for private firms, thereby making ratios based on them meaningless. Even when positive, earnings and EBITDA have higher variance, leading to less stable valuation ratios. Furthermore, PE firms often compute adjusted EBITDA or earnings measures to focus on the recurring earnings of the private companies. These adjustments could be subjective and unobservable, thus lacking comparability across transactions.

Since private firms with negative or zero sales are already excluded, the distribution of the log **P/S** ratio is very close to a Gaussian distribution, as shown in Figure 8. However, the raw distribution is not a Gaussian and resembles an exponentially decaying series. Also note that, because of the exclusion of outliers, the log transformation produces thinner tails when compared to a Gaussian distribution.

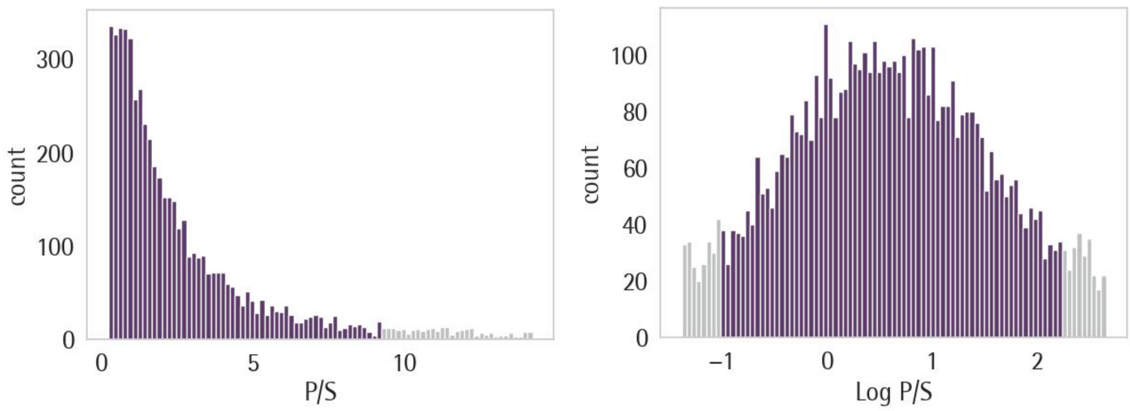
## 4.2 Explanatory Variables

Based on the literature review in Chapter 3, the below variables are proposed to explain the observed valuation ratios of private companies. These measures relate to key financial characteristics of the private companies and the nature of private markets, which private equity firms are likely to intuitively consider in their valuation of the asset.

1. **Size** is measured as the logarithm of sales. Typically, in the literature, size can be measured as the total value of book assets, total sales, or as market capitalisation (or enterprise value), with all these measures being highly correlated Dang et al. (2018). However, the market capitalisation of private companies is unavailable. Also, when modelling **P/S** based on observed transactions, enterprise value cannot be an independent predictor as it is partially captured in the **P/S** ratio. Moreover, due to data paucity issues, sales is more commonly observable than the book value of assets, as often primary data vendors do not have access to the entire financials of a transacted private company. Thus, for empirical reasons, sales is used as a proxy for size. Interested readers can refer to Dang et al. (2018) for a discussion on using sales as a proxy for size and its merits and demerits.
2. **Book leverage** is expected to be related to discount rates and is measured as the logarithm of a constant plus the total debt in the financial statements of the private company divided by its total sales. These specific transformations ensure that the predictor variable resembles a Gaussian distribution. For examining a nonlinear relationship between valuation and leverage, it is also experimented with the square of this measure.
3. **Growth** measured as the rate of growth in revenue of the private company, is expected to affect firm valuation. Measuring growth conventionally as in the finance literature, such as the ratio of market to book value of assets or as Tobin's $q$ that is computed as the ratio of market value of assets to its replacement costs, is not feasible for private companies that do not have regularly observable market prices. However, sales growth is easily observable and an appropriate proxy for the growth opportunities of the firm.
4. **Profitability** is measured as the ratio of EBITDA to sales. EBITDA is available more commonly for private companies and moreover is also independent of its capital structure, thereby making EBITDA margin comparable across private firms with different levels of debt compared to net profit. In instances where another measure of profit is available such as EBIT (Earnings Before Interest and Taxes), operating profit, or net profit, adjustment factors are computed that correspond to the median percentage difference between the alternative measure and EBITDA, using the subsample of positive profits. These adjustment factors are then applied to the absolute value of the profit measures to get the adjusted imputed **profitability**.[3]

3 - Adding back the product of the absolute value of the profit measure and the adjustment factor to the former measure, ensures that the direction of adjustment is the same whether the company has positive or negative profits. In general, operating profit is greater than EBITDA, which is greater than EBIT, which in turn is greater than Net Income. The imputations are designed in such a manner that this hierarchy is preserved irrespective of whether the company has a positive or negative value for the profitability measure.

Figure 8: Distribution of **P/S**




5. **Market valuation** factor is likely to be associated with the required return for private companies as the latter are exposed to the same economic fundamentals as listed public equities. The **Market valuation** factor is measured as the logarithm of the **P/S** ratio of the value-weighted CRSP index (The Centre for Research on Security Prices), which includes all firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that are designated as common shares. Since the modelled variable is a log valuation ratio, it is preferable to measure the market factor in a similar manner. Moreover, the **market valuation** factor is orthogonalised by regressing the **log (P/S)** market measure on the contemporaneous **log (P/S)** measure for the asset's activity (i.e., **industry valuation** factor), and use the residuals. This step ensures that there are fewer correlated predictors and the **market** factor captures variation in valuation that is not attributable to industrial activity level trends.
6. **Industry valuation** factor is likely to be associated with the required return for private companies as both public and private companies operating in the same activity face similar economic fundamentals and demand-supply equation. The **industry valuation** factor is computed as the logarithm of the **P/S** ratio of the subset of stocks in the value-weighted CRSP index operating in the same PECCS™ activity subclass. It is quite reasonable to expect the valuation of private companies to closely correlate with the valuation of public stocks that operate in the narrow industrial activity as the focal private company.
7. **Age** is measured as the logarithm of a constant plus the difference between the transaction year and the year in which the private company was founded and can be associated with required returns due to the availability or nonavailability of performance history.
8. **Labour intensity**, measured as the logarithm of the number of employees of the firm divided by its sales, can be expected to be associated with required returns due to the potential for restructuring and the level of coordination costs and incentive structures at the private firm.
9. **Market share** of an asset can be associated with the required return through the effect of competition on business sustainability. However, computing market share is complicated by a lack of reliable data on the universe of private companies in any given activity. To overcome this data paucity, market share is expressed on the basis of the combined sales of all stocks operating in the PECCS™ activity class and the focal asset. Specifically, **market share** is computed as the logarithm of a constant plus the proportion of revenue of a private

company to the sum of the total revenue of all public firms in the same PECCS™ activity class and the private company's revenue.

10. **Patent** is measured as an indicator variable that takes the value of one for private companies that have one or more active or pending patents. An indicator is preferable as the distribution of patents in the sample is skewed, with fewer than 26.2% of the sample companies having one or more patents. This is not surprising as, even among publicly listed companies, the distribution of patents is very asymmetric, with the majority of public companies having zero patents (e.g., (Fang et al., 2014)).
11. **Deal leverage**, incurred as part of the transaction, can also affect the required returns of a private company, both by way of signalling private information about the return prospects of the company and through the traditional tax savings and bankruptcy tradeoffs. **Deal leverage** is measured as the logarithm of a constant plus the ratio of the debt incurred during the observed transaction to the company's sales.
12. **Herding** by investors in private markets can also affect valuations and hence required returns, as the lowered cost of capital in the sector, increased availability of assets for benchmarking, and the potential higher level of investment opportunities in the activity (a source of herding) can influence the valuation of target assets. **Herding** is measured as the proportion of deals in the target company's PECCS™ activity to the overall sample deals in the year prior to the transaction. To specifically capture the component of capital inflows into private assets, the amount of **dry powder** computed as the logarithm of the annual dry powder with private equity funds is included. **Dry powder** represents the amount of committed but uncalled capital. **Dry powder** is standardised by dividing with the focal company's sales to capture the perspective of how capital availability can influence an individual transaction with respect to the company's size.
13. **PE backing** is an indicator variable that takes the value of one when the existing ownership structure of the private firm includes at least one private equity owner. This variable will capture transactions that are already professionally structured or managed and hence can command a different valuation, such as GP-led secondaries.
14. Private equity returns and hence private company valuations are highly sensitive to the cost of debt financing. The cost of debt is proxied as the logarithm of a constant plus the long-term **interest rate** in the country where the private company is headquartered. In addition, to the interest rate, **term spread** is also included as a predictor, measured as the logarithm of constant plus the difference between the 20-year and three-month rate on government securities in the country of the private company. Both these measures are able to additionally capture the economic differences across assets in different countries, and also act as a proxy for the cost of financing and economic prospects. For example, a higher term spread value indicates investors' belief in a future stronger economy, and vice-versa.
15. For identifying other transaction characteristics, we include indicators for whether the target was a public company, **public co**, whether the deal is structured as an add-on transaction, i.e., acquired by another private equity portfolio company, **add-on**, and the percentage of ownership sought in the transaction, **control**.
16. To incorporate additional market characteristics, we include the logarithm of a constant plus the **VIX index** published by the Chicago Board Options Exchange to capture investor risk attitudes and **public market volatility**, measured as the standard

deviation of CRSP value-weighted indices monthly returns.

We also include investor preferences for growth over value (**value factor**), small over large stocks (**size factor**), high recent profits over high recent losses (**momentum factor**), highly profitable versus less profitable (**profitability factor**), and conservatively investing firms over aggressive investing firms (**investment factor**). These five Fama-French factors explain the cross-section of public stock returns quite well Fama and French (2015). The five public market factors are obtained from the Kenneth French website.

In addition, we include measures of stock market liquidity. These are measured as the Amihud's price impact (Amihud and Mendelson, 1986). The growing literature on stock liquidity also points to the differences between price impact and trading volume, and thus a modified Amihud's measure is also computed, where the numerator is a constant instead of the periodic return. Thus, both **market price impact** and **market trading volume** are included as predictors. In addition, we include **industry price impact**, which is measured based on the stocks that are in the same PECCS™ activity class as the focal private company.

Finally, we construct a private companies based size factor **private size factor** as the difference in **P/S** between smaller and larger private companies in each quarter, where size is defined by sample median of sales (i.e., assets with below median sales in sample forms part of the small group). This factor is likely to capture the time-varying private capital market investors' preference for size, which may play a role in the valuation of a company.

17. To capture macroeconomic characteristics that prevail at the time of the private company transaction, we include the **GDP growth**, **CPI growth**, **forex change** (defined as change in local currency in the previous 12 months with respect to the US$) and an indicator for whether the private company is headquartered in an emerging economy or not (based on World Bank definitions of countries).
18. To further capture sector/activity level differences, we include a measure of industry concentration (Herfindahl of industry level sales of public firms), **industry concentration**, and a text-based similarity of the private company with the till transaction date sample of private companies. **Similarity** is computed in a manner similar to Hoberg and Phillips (2016) based on business descriptions of the private company. An indicator variable **hitech** that identifies assets that perform hitech activities is included. Specifically, any company that operates in an industry that is a combination of a traditional sector with technology is assigned a value of one for this indicator, e.g. fintech, insurtech, artificial intelligence, etc.
19. In addition to the above variables, we include indicator variables for various PECCS™ pillars consisting of the 12 classes of activities, the three classes of lifecycle phases, the four classes of revenue model, the two classes of customer model, and the three classes of value chain.

Also in constructing all the above variables, we ensure that they are based on the latest information that is available prior to the transaction (i.e., without any look-ahead bias), and hence may proxy for the information available with dealmakers.

## 4.3 Summary Statistics

Table 9 shows descriptive statistics for the explanatory variables discussed above. The distribution of the explanatory variables (only continuous variables) and pairwise correlations between the key variables are presented in the Appendix. The plots show that the explanatory variables are largely uncorrelated. To further formally rule out multicollinearity problems in regressions that may bias the standard errors and lead to incorrect inferences, we compute the variance inflation factors for the explanatory variables in subsequent sections.

Additionally, in the Appendix, we plot the **P/S** ratios of the sample by deciles of a few key explanatory variables. The **P/S** decreases somewhat monotonically with **size** and increases with **deal leverage**, but with other measures, there are no clear monotonic trends.

In subsequent analysis, when required, we add constants to logarithmic transformations or make additional transformations to ensure explanatory variables follow a distribution close to a Gaussian one to make them more compatible with regressions. For the sake of interpretation, in all such transformations the direction of relationship between transformed and original variable is maintained, i.e., both are positively correlated (unless stated explicitly as *inverse*).

Table 9: Sample descriptive statistics

| Variable | Obs | Mean | StdDev | Min | Median | Max |
|---|---|---|---|---|---|---|
| P/S* | 5,438 | 2.76 | 2.73 | 0.25 | 1.78 | 14.21 |
| Key explanatory variable | | | | | | |
| Size* | 5,438 | 463.9 | 1,760.3 | 5.1 | 96.6 | 37,818.1 |
| Growth* | 5,438 | 0.47 | 7.04 | -0.96 | 0.06 | 311.51 |
| Profitability | 5,438 | 14.48 | 26.89 | -142.60 | 12.84 | 127.64 |
| Book leverage* | 5,438 | 0.61 | 4.10 | 0.00 | 0.00 | 161.08 |
| Market valuation* | 5,438 | 1.26 | 0.39 | 0.48 | 1.15 | 2.36 |
| Industry valuation* | 5,438 | 1.82 | 1.67 | 0.10 | 1.28 | 12.29 |
| Term spread* | 5,438 | 0.02 | 0.01 | -0.05 | 0.02 | 0.08 |
| Optional explanatory variable | | | | | | |
| Age* | 5,438 | 35.40 | 34.41 | -1.00 | 23.00 | 214.00 |
| Patent | 5,438 | 0.26 | 0.44 | 0.00 | 0.00 | 1.00 |
| Market share* | 5,438 | 0.28 | 1.38 | 0.00 | 0.04 | 51.72 |
| Deal leverage* | 5,438 | 0.21 | 0.77 | 0.00 | 0.00 | 11.23 |
| Herding* | 5,438 | 0.15 | 0.10 | 0.00 | 0.12 | 0.42 |
| PE back | 5,438 | 0.62 | 0.49 | 0.00 | 1.00 | 1.00 |
| LT rate* | 5,438 | 0.03 | 0.02 | -0.01 | 0.03 | 0.16 |
| Industry concentration* | 5,438 | 1,186.4 | 1,101.8 | 222.9 | 857.3 | 7,931.4 |
| Forex change | 5,438 | 0.00 | 0.07 | -0.43 | 0.00 | 0.33 |
| VIX* | 5,438 | 18.32 | 6.95 | 10.13 | 16.68 | 62.67 |
| GDP growth | 5,438 | 0.03 | 0.08 | -0.34 | 0.04 | 0.43 |
| CPI growth | 5,438 | 0.02 | 0.02 | -0.02 | 0.02 | 0.30 |
| Labour intensity* | 5,438 | 13.89 | 34.57 | 0.04 | 5.40 | 423.47 |
| Control | 5,438 | 0.71 | 0.31 | 0.01 | 0.80 | 1.00 |
| Addon | 5,438 | 0.17 | 0.37 | 0.00 | 0.00 | 1.00 |
| Public co | 5,438 | 0.06 | 0.23 | 0.00 | 0.00 | 1.00 |
| Similarity | 5,438 | 0.12 | 0.02 | 0.06 | 0.11 | 0.27 |
| Size factor | 5,438 | 1.41 | 9.07 | -25.37 | 0.32 | 54.00 |
| Value factor | 5,438 | -1.22 | 13.40 | -35.01 | -1.85 | 73.84 |
| Momentum factor | 5,438 | 1.40 | 14.70 | -56.40 | 2.96 | 54.93 |
| Profitability factor | 5,438 | 2.81 | 7.73 | -37.24 | 2.32 | 58.31 |
| Investment factor | 5,438 | 0.63 | 7.41 | -11.01 | -0.86 | 44.82 |
| Size private factor | 5,438 | 1.35 | 0.64 | -0.42 | 1.32 | 3.10 |
| Dry powder* | 5,438 | 7.12 | 3.66 | 2.46 | 6.42 | 20.83 |
| Market volatility* | 5,438 | 0.04 | 0.02 | 0.01 | 0.04 | 0.09 |
| Market price impact* | 5,438 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

* indicates the variables, when used in regressions, are subject to log or other transformations

Sample descriptive statistics: continued

| Variable | Obs | Mean | StdDev | Min | Median | Max |
|---|---|---|---|---|---|---|
| Sector price impact* | 5,438 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hitech | 5,438 | 0.23 | 0.42 | 0.00 | 0.00 | 1.00 |
| Emerging country | 5,438 | 0.03 | 0.17 | 0.00 | 0.00 | 1.00 |
| PECCS™ indicators | | | | | | |
| Activity Education & public | 5,438 | 0.01 | 0.11 | 0.00 | 0.00 | 1.00 |
| Activity Financials | 5,438 | 0.07 | 0.26 | 0.00 | 0.00 | 1.00 |
| Activity Health | 5,438 | 0.06 | 0.24 | 0.00 | 0.00 | 1.00 |
| Activity Hospitality & ent. | 5,438 | 0.07 | 0.26 | 0.00 | 0.00 | 1.00 |
| Activity Information & comm. | 5,438 | 0.18 | 0.38 | 0.00 | 0.00 | 1.00 |
| Activity Manufacturing | 5,438 | 0.32 | 0.47 | 0.00 | 0.00 | 1.00 |
| Activity Natural resources | 5,438 | 0.03 | 0.17 | 0.00 | 0.00 | 1.00 |
| Activity Professional & services | 5,438 | 0.09 | 0.29 | 0.00 | 0.00 | 1.00 |
| Activity Real estate & const. | 5,438 | 0.03 | 0.17 | 0.00 | 0.00 | 1.00 |
| Activity Retail | 5,438 | 0.05 | 0.23 | 0.00 | 0.00 | 1.00 |
| Activity Transportation | 5,438 | 0.05 | 0.23 | 0.00 | 0.00 | 1.00 |
| Activity Utilities | 5,438 | 0.02 | 0.15 | 0.00 | 0.00 | 1.00 |
| Lifecycle Mature | 5,438 | 0.63 | 0.48 | 0.00 | 1.00 | 1.00 |
| Lifecycle Growth | 5,438 | 0.29 | 0.46 | 0.00 | 0.00 | 1.00 |
| Lifecycle Startup | 5,438 | 0.08 | 0.27 | 0.00 | 0.00 | 1.00 |
| Revenue Model Production | 5,438 | 0.68 | 0.47 | 0.00 | 1.00 | 1.00 |
| Revenue Model Advertising | 5,438 | 0.05 | 0.22 | 0.00 | 0.00 | 1.00 |
| Revenue Model Reselling | 5,438 | 0.15 | 0.35 | 0.00 | 0.00 | 1.00 |
| Revenue Model Subscription | 5,438 | 0.13 | 0.33 | 0.00 | 0.00 | 1.00 |
| Cust. Model Consumer focused | 5,438 | 0.36 | 0.48 | 0.00 | 0.00 | 1.00 |
| Cust. Model Business focused | 5,438 | 0.64 | 0.48 | 0.00 | 1.00 | 1.00 |
| Value Chain Products | 5,438 | 0.47 | 0.50 | 0.00 | 0.00 | 1.00 |
| Value Chain Services | 5,438 | 0.46 | 0.50 | 0.00 | 0.00 | 1.00 |
| Value Chain Hybrid | 5,438 | 0.07 | 0.26 | 0.00 | 0.00 | 1.00 |

* indicates the variables, when used in regressions, are subject to log or other transformations

**Full names of shortened variables above are available in Table 6.

# 5. The Empirical Approach

This chapter provides details on the empirical methodology. Table 10 presents the summary, which illustrates the sequential nature of the methods and the purpose of each step. In the subsequent sections, we provide more details on the key methods including subset selection (forward stepwise), Lasso regressions, and dynamic linear models.

## 5.1 Subset Selection

Having several potential factors does not guarantee an optimal model. Although adding numerous factors can reduce a regression's residual sum of squares (or increase its $R^2$), it might introduce other econometric issues such as multicollinearity. Thus, taking advantage of statistical approaches designed to identify a subset of the predictors that will be related to valuation, we estimate several least square models (Gareth et al., 2013).

To select the best subset of regressors, one approach is to estimate several least squares regressions for each possible combination of the predictor variables. For example, if there are ten predictors, one can estimate ten single-variable models (i.e., choosing one variable from ten). Next, one can estimate $\binom{10}{2}$, i.e., 45 models based on all combinations of two predictors. This process can be repeated for models ranging from 1 to 10 variables. From the result of all the estimation, one can then identify the model that best fits the data, based on some model selection criteria such as Adjusted $R^2$ or the Bayesian Information Criterion (BIC) or the Akaike Information Criterion (AIC) of all the models.

But in this approach, the number of potential variables is much greater than 10 which increases the number of models to be estimated tremendously. For example, with 25 variables, the above approach requires the estimation of 33.55 million models, which is very intensive in terms of computational resources and time requirements. Thus, we implement a forward stepwise selection algorithm to select the best model for each number of predictors (e.g., Hastie et al., 2017).

Forward stepwise selection approach starts from a model without any variables and adds one variable at a time to improve the model. At every step, we select the variable that is the most significant based on whether it reduces the residual sum of squares the most, or gives the highest increase in $R^2$, or has the smallest $p$-value. The algorithm basically helps to reduce the number of models to be estimated at each step, thus improving over the brute-force method of estimating all possible regressions.

## 5.2 Lasso Regressions

Although forward stepwise selection allows one to select the best linear model given the potential set of regressors, it is also possible that the regressors are related to valuation in a nonlinear manner. For examining that, one can estimate polynomial specifications of regressor combinations, and feature selection (i.e., narrowing down the list of potential variables) becomes a larger problem. When second order combinations of regressors are allowed, then the additional information from such variables to valuation is likely to be lower, i.e., low signal-to-noise ratios, or in other words marginal information from such variables with regards to valuation is lower. In such scenarios, prior work has found that Lasso feature selection regressions perform well (e.g., Hastie et al., 2017).

Table 10: Econometric framework

| Step | Detail | Purpose |
|---|---|---|
| 1 | Split regressors into required and optional | Designate some regressors like size, leverage, market valuation, etc., as these variables inarguably determine private company valuation without requiring statistical justification |
| 2 | Forward stepwise selection | Estimate the list of optional regressors that are key for valuation |
| 3 | Polynomial Specifications | Examine whether there are non-linear relationships between regressors and valuation |
| 4 | Lasso feature selection | Among the second-degree polynomial predictors (i.e., interactions and squared regressors), determine the key predictors |
| 5 | Principal Component Analysis (PCA) | Distill the selected second-order predictors into a few principal components that are economically intuitive as well |
| 6 | Dynamic Linear Models (DLM) | For the model that includes required, selected optional, and principal component second-order variables as regressors, estimate time-varying coefficients |

The Lasso method (Least Absolute Shrinkage and Selection Operator) regularises model parameters by shrinking the regression coefficients, reducing some of them to zero. The feature (or variable) selection phase occurs after the shrinkage, where every non-zero value is selected to be used in the regression model. The Lasso method is significant in the minimisation of prediction errors that are common in statistical models. The objective function in Lasso is to minimise the below objective function:

$$\left[\sum_{i=1}^{N} Y_i - \beta_0 - \sum_{k=1}^{K} \beta_k X_{k,i}\right]^2 + \alpha \sum_{k=1}^{K} |\beta_k| \quad (5.1)$$

where $i$ refers to the private company valuation $Y_i$, $k$ refers to the potential regressor $X_k$, $\beta$ corresponds to the coefficients that need to be estimated to relate the regressors with valuation, and $\alpha$ refers to a parameter that imposes a penalty for the coefficient.

The first squared term in Equation 5.1 corresponds to the sum of the squared errors, the typical objective function in an ordinary least-squares regression. The second part is the penalty imposed for the coefficient which increases with the value of coefficient.[1] Thus, by specifying a value of $\alpha$ or hypertuning the parameter using the sample, Equation 5.1 can be minimised for a level of $\alpha$ and set of regressors whose coefficients in the regularisation step is still not zero.[2]

## 5.3 Dynamic Linear Models

In this section, the approach to estimating time-varying factor prices in the private company valuation model is described.

The simple linear model ignoring the second order features selected by Lasso regressions that explain the relationship between valuation ratios and potential factors can be expressed as below:

$$Y_{i,t} = \beta_1 + \sum_{k=2}^{K} \beta_k X_{k,i,t} + \varepsilon_t \quad (5.2)$$

where $Y_{i,t}$ corresponds to a valuation ratio of asset $i$ at time $t$ with $k$ characteristics/factors $X_{i,t}$ such as size, leverage, etc. Here $\beta_{k,t}$

1 - Note that Lasso regressions are implemented by standardising the variables, and hence larger coefficient value corresponds to more important variables.

2 - Also, in Lasso regression estimation a k-fold cross validation approach is followed, that splits the sample into training ($\frac{k-1}{k}$%) and test samples ($\frac{1}{k}$%). The k-fold approach maximises the sample by not fixing a training sample, and adopting an iterative approach where a small fraction is designated as test successively.

measures the price of each factor and is estimated in an unbiased manner when the error term $\varepsilon_t$ is independently and normally distributed. However, in private asset markets neither could be assumed, with the error terms likely to capture systematic valuation errors, such as periods when larger or smaller firms may be preferred more, and hence overvalued. In other words, investor preferences and market conditions may evolve, which may alter the price of factors over time. This will lead to the error term systematically capturing these trends, leading to non-normally distributed errors, and hence biased estimates.

Thus, the coefficients or $\beta$ are allowed to evolve over time to accommodate for time varying factor prices, as shown below:

$$Y_{i,t} = \beta_{1,t} + \sum_{k=2}^{K} \beta_{k,t} X_{k,i,t} + \varepsilon_t \tag{5.3}$$

The errors above can be assumed to be independent and normally distributed, as the evolution of $\beta$ can capture time-varying investor preferences for private company characteristics. The $\beta$ can be modelled as a first-order autocorrelated process as below:

$$\beta_{k,t} = \beta_{k,t-1} + w_{k,t} \tag{5.4}$$

Defining $\theta_t$ as a vector of factor prices $\beta_{1,t}, \beta_{2,t}, ...\beta_{K,t}$, one can rewrite Equation 5.3 as:

$$\begin{aligned} Y_{i,t} &= X_t^{\prime}\theta_t + v_t \\ theta_t &= G_t\theta_{t-1} + W_t \end{aligned} \tag{5.5}$$

where $v_t$ is the variance or noise of the pricing equation and is independent and identically distributed, and $W_t$ is the co-variance matrix of the model's coefficients.

Equation 5.5 is a state-space Markov chain model between observable valuation and autoregressive vector of factor prices. When $G_t$ is the identity matrix, the regression coefficients can be thought of as being independent random walks, i.e., each factor's effect on valuation is serially correlated but the different factors themselves are independent of each other. However, if that is not the case like when factors jointly evolve and affect value, for example, say the effect of size and leverage on valuation changes jointly over time, the covariance matrix $W_t$ can capture such dynamics, which are not feasible in the linear model proposed in Equation 5.2.

Equation 5.5 is estimated using Bayesian techniques that can estimate $K$ coefficients each time a new observation is available, which in this setting, is a new transaction in private companies, thus allowing one to learn and update unbiased estimates of factor prices, each time a new biased and noisy transaction is observed. For further information, please refer to Blanc-Brude and Tran (2019) which provides details of the estimation of the dynamic linear model using sparsely observed transaction data.

# 6. Factor Estimation Results

This chapter provides the estimation of the factors that affect the valuation of private companies based on the empirical methodology laid out in Chapter 3.

## 6.1 Ordinary Least Squares Regressions

Estimating the factor model using ordinary least square (or OLS) regressions has several shortcomings:

First, by pooling the data, OLS regressions assume each observation is independent. However, valuations in private company transactions are in general serially correlated. For example, the Ljung-Box test for autocorrelation of sample transformed **P/S** values has a $\chi^2$ test statistic of 69.2 with a *p*-value of less than 0.01, rejecting the null hypothesis of no autocorrelation at even 1% level of significance.

Second, correlations between transformed **P/S** and explanatory variables also vary with time. Thus, OLS regressions estimate only an average effect of a factor across the entire sample, whereas the relationship between factors and valuations are varying with investor preferences and learning.

### 6.1.1 Simple OLS Model

Despite these shortcomings, OLS regressions provide an initial view of the association between the factors and **P/S** ratio which can be intuitively interpreted. Before proceeding to regress the variables using an OLS model, first the variables are categorised as either required and optional variables. The required variables are inarguably associated with valuation, and statistical justification is not needed for their inclusion. The variables deemed as required include: 1) **size**, 2) **growth**, 3) **profitability**, 4) **book leverage**, 5) **market valuation**, 6) **industry valuation**, 7) **term spread** and all the PECCS™ indicators. The subset algorithm, the method to find out which of the optional variables are important, is not allowed to select among the required variables, and is instead run only on the remaining variables that are reported in Table 9. The results of OLS regressions with only the required variables are presented in Table 11.

The results indicate that transactions in private companies happen at higher valuations for smaller, more profitable, and highly leveraged firms. Also, deals that happen when public markets, and especially the stocks engaged in similar industrial activities, are valued higher and term spreads are narrower, leading to a higher valuation of the private company. The coefficients for these variables are significant at least at the 5% level, indicating these variables have statistically significant effects on the valuation. These findings are fairly consistent with expectations, with the exception of leverage, on which the prediction was ambiguous. The data indicates that private companies that have incurred more debt are valued higher, pointing towards the signalling effect of leverage, i.e., better assets can support higher borrowings, assuming creditors scrutinise borrowers adequately.

In terms of PECCS™ indicators, for each pillar one class is dropped from the regression. Hence the interpretation of each coefficient in every pillar is a relative effect with respect to the omitted class. In terms of activity, when compared to **manufacturing**, **health**, **financials**, and **natural resources** are valued significantly higher, whereas **retail** and **hospitality**

& **entertainment** sectors are valued significantly lower. Regarding the lifecycle phases of assets, unsurprisingly companies in the **startup** phase are valued signficantly higher, while **growth** companies are marginally highly valued, when compared to companies that are categorised as **mature**. Concerning revenue models, compared to **production** based revenue models, **subscription** based models are valued at a premium. Also, companies that operate a **consumer focused** customer model exceed valuations of **business focused** companies. Finally, companies that have **services** type value chains are valued greater than **products** type value chains.

### 6.1.2 Subset Selection

The problem of selecting the best subset of regressors for **P/S** is not trivial (Gareth et al., 2013). For each number of regressors, the subset selection algorithm identifies the regression using variables that generate the lowest residual sum of squares (or highest $R^2$). Starting from zero variables (i.e., just the mean), this procedure is repeated by increasing the number of variables by one. Thus, the implementation of the subset selection algorithm yields a series of best OLS models for each number of variables, i.e., 1, 2, 3 until the maximum number of variables is used. However, such broad subset selection algorithms might be difficult to implement when the number of potential regressors is numerous. Thus, a forward-stepwise subset selection is implemented, which can be quickly estimated and provides similar results to a brute force (i.e., all possible combinations of regressors) subset selection approach.

The objective of the forward-stepwise subset selection algorithm is two-fold: first, find the best set of regressors for each number of variables that is preferred and, second, generate model choice metrics that can allow identification of the optimal number of regressors. Implementing the forward-stepwise subset selection over the optional variables, the best models for each number of preferred optional variables are determined, the results of which are not reported for the sake of brevity. Using the best model chosen by the algorithm for each number of optional variables, in Figure 9, the model performance for this subset of best models is presented. Specifically, the residual sum of squares (RSS), adjusted $R^2$'s, Akaike's Information Criteria (AIC), and Bayesian's Information Criteria (BIC) are presented against the number of optional variables included in the model.

Prioritising model sparsity, the best model as indicated by BIC is used, as BIC is a well-known to favour models that are parsimonious. Specifically, BIC penalises the model for the number of parameters more than AIC does (Schwarz, 1978; Raftery, 1995). Thus, the best model as indicated by BIC is the one with nine optional variables included. However, in subsequent analysis, even if models based on AIC or adjusted $R^2$'s are chosen, results are not materially different.

### 6.1.3 Polynomial Specifications

Next, we explore whether the predictors might be related in a nonlinear manner with private company valuation. Rather than pick and choose variables that may exhibit nonlinear relationships, we experiment with all potential polynomial specifications to check if overall model performance is improved. To maintain the simplicity and intuitiveness of the model, we restrict the experimentation to second order polynomial terms that include all second-order terms of predictor variables as described below.

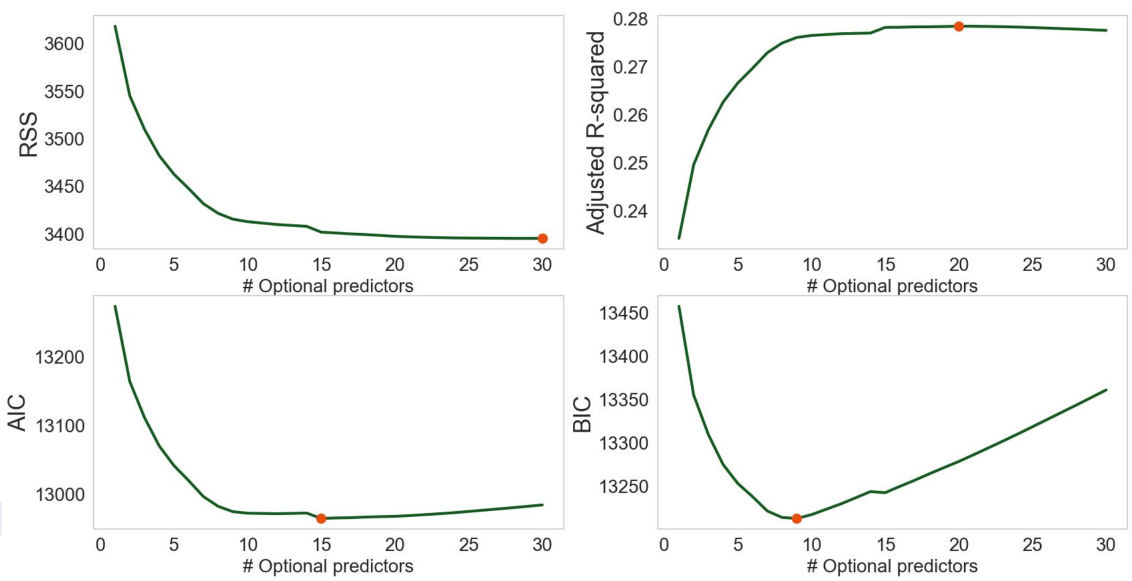
To implement this approach, first all combinations of predictor variables in the second degree are created. That is for any two regressors $a$ and $b$, a list of regressors that include $a$, $b$, $a^2$, $b^2$, and $ab$ is created. Using this expanded set of regressors, we can implement a forward stepwise subset selection. Using the

Table 11: Ordinary least squares regression

| | Dependent variable: P/S | |
|---|---|---|
| Explanatory variables | Estimate ($t$-statistic) | Std Error |
| Intercept | 3.246** (2.59) | 1.255 |
| Size | -0.102*** (-11.23) | 0.009 |
| Growth | 0.005 (0.18) | 0.028 |
| Profitability | 0.007*** (15.37) | 0.000 |
| Book leverage | 0.065*** (6.78) | 0.010 |
| Market valuation | 0.067*** (5.83) | 0.011 |
| Industry valuation | 0.374*** (10.55) | 0.035 |
| Term spread | -3.909** (-2.19) | 1.783 |
| PECCS™ indicators | | |
| Activity Education & public | 0.073 (0.64) | 0.114 |
| Activity Financials | 0.151** (2.05) | 0.073 |
| Activity Health | 0.175*** (2.87) | 0.061 |
| Activity Hospitality & entertainment | -0.163** (-2.13) | 0.077 |
| Activity Information & communication | 0.079 (1.29) | 0.061 |
| Activity Natural resources | 0.282*** (4.02) | 0.070 |
| Activity Professional & other services | -0.112* (-1.68) | 0.066 |
| Activity Real estate & construction | 0.054 (0.63) | 0.085 |
| Activity Retail | -0.202*** (-2.95) | 0.068 |
| Activity Transportation | -0.118 (-1.55) | 0.077 |
| Activity Utilities | -0.107 (-1.12) | 0.096 |
| Lifecycle Growth | 0.056** (2.16) | 0.026 |
| Lifecycle Startup | 0.220*** (5.02) | 0.044 |
| Revenue Model Advertising | 0.036 (0.63) | 0.058 |
| Revenue Model Subscription | 0.140*** (3.16) | 0.044 |
| Revenue Model Reselling | -0.005 (-0.11) | 0.049 |
| Customer Model Consumer focused | 0.105*** (3.70) | 0.028 |
| Value Chain Services | 0.114** (2.24) | 0.051 |
| Value Chain Hybrid | 0.341 (4.69) | 0.073 |
| Observations | 5,438 | |
| Adjusted $R^2$ | 0.205 | |

Variables are transformed as indicated in Table 9

Figure 9: Optimal model for each set of variables




best model chosen by the algorithm for each number of optional second-order variables, Figure 10 presents the model performances. The model performance has drastically improved from the best linear models. For example, the best BIC indicated model that includes one optional variable has an adjusted $R^2$ of 0.330, much above the linear model in the previous subsection of 0.205. Thus, incorporating second order terms can yield a better model.

### 6.1.4 Lasso Regressions

A potential problem with this approach is the increased complexity of the models with a small increase in the number of predictors. For example, with a list of 48 continuous variables, all combinations of predictors can generate $\frac{48\times(48-1)}{2} = 1,232$ regressors. Such a large set of regressors gives rise to concerns of overfitting, that is the model fitting the in-sample data extremely well, but performing very poorly in out of sample predictions.

The objective of this paper is to develop a parsimonious model that can estimate the valuation of any private company given its characteristics at any given point in time. Thus, to preserve the generalisability of the model, we implement Lasso regressions on the second order terms to identify the key factors that are important in the study. As discussed in Chapter 3, Lasso or least absolute shrinkage and selection operator provides the best and a smaller subset of regressors that capture as much variation as possible in the dependent variable.

First, based on all second order terms of the entire list of predictors, all predictors are standardised. Next, we estimate the objective function of the Lasso regression. We experiment with various values of $\alpha$, the penalty parameter that penalises the objective function for the inclusion of more regressors. The relationship between the coefficients of all standardised regressors and $\alpha$ is depicted in Figure 11. As the value of $\alpha$ is increased, less and less of the regressors' coefficients are non-zero. Note that $\alpha$ of zero corresponds to the OLS equivalent regression using all second order terms. Similarly, a really large value of $\alpha$ drops all the regressors. Thus, by specifying an appropriate level of $\alpha$ between zero and a high value, one can narrow down to a smaller potential set of regressors.

To determine the optimal value of alpha, i.e., hyperparameter tuning, we perform a $k$-fold

Figure 10: Optimal model with polynomial degree two variables

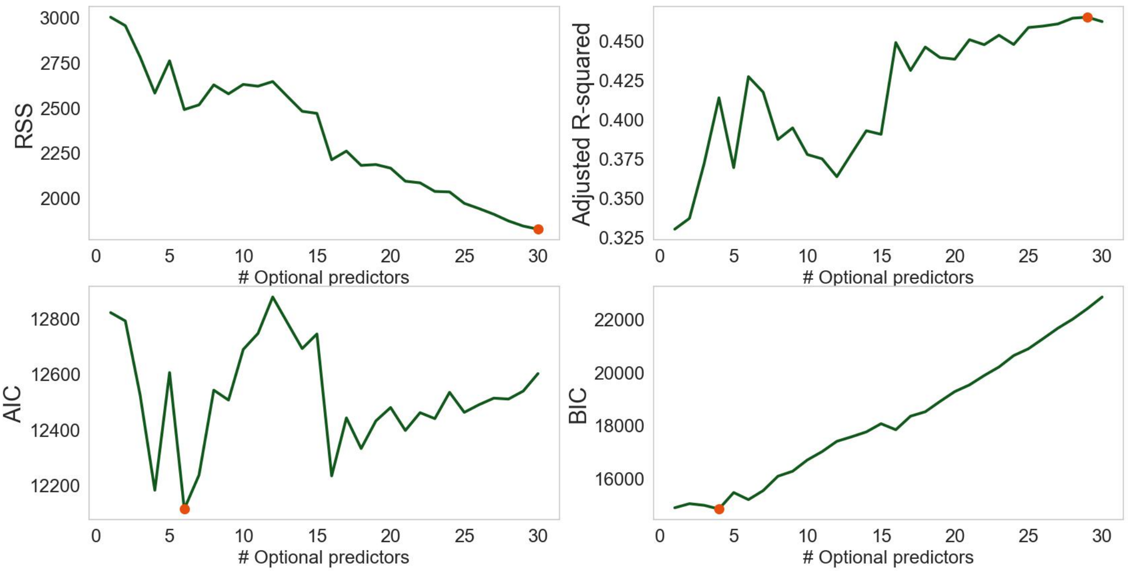
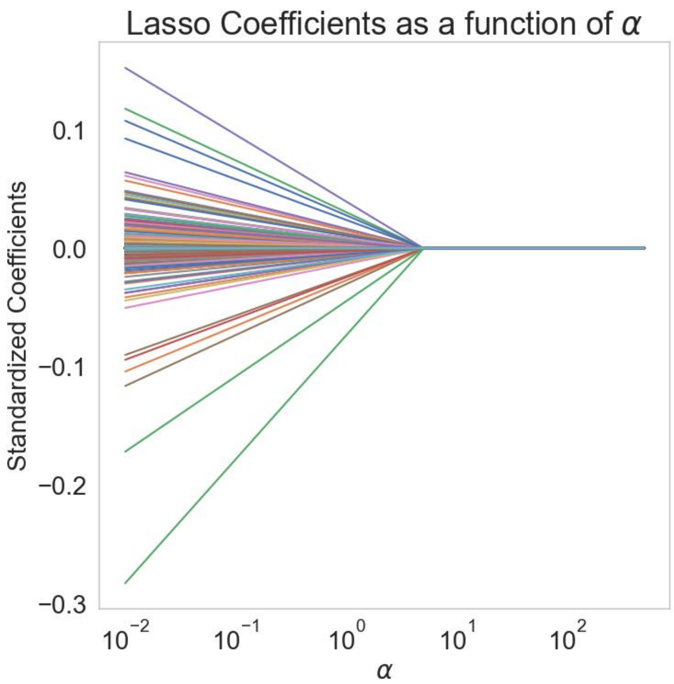



Figure 11: Role of $\alpha$ in Lasso regressions



cross validation approach. This approach splits the sample into a train and test dataset, and for any value of $\alpha$ fits a model on the training dataset, and tests the fitted model's predictions on the test dataset. An advantage of the $k$-fold cross validation approach is that it is an iterative approach that dynamically splits the dataset into train and test, maximising the sample, rather than a static split of the sample where the test sample is never part of training data. Specifically, the $k$-fold cross validation approach splits the data into $k$ parts, and performs the exercise $k$ times where each $1/k$ portion of the dataset is designated as the test dataset.

$k$ is chosen to be 5, thus designating 20% of the data as test each time, and this approach finds an optimal $\alpha$ of 0.0043. The average mean squared errors of the models estimated in this approach as a function of $\alpha$ is plotted in Figure 12. The mean-squared errors in each fold are depicted as a function of $\alpha$. The optimal $\alpha$ is chosen such that it minimises the mean squared errors averaged across all the five folds.

With optimal $\alpha$ determined, the focus next is on the list of regressors that are determined to be important. Table 12 presents selected regressors, with a deeper (lighter) shade of green indicating a more (less) important predictor. The first column in the Table shows the regularised coefficient at the optimal $\alpha$ and can be directly interpreted as how important the predictor is. Also, one can observe from the variables, that some of the predictors occur in multiple rows. For example, **industry valuation**, is part of three regressors. Thus, a potential problem with including all these polynomial regressors in the model chosen by the forward-stepwise method is an increase in collinearity among the regressors. Moreover, adding ten more regressors increases model complexity. Thus, other econometric techniques are explored to reduce the set of regressors further.

### 6.1.5 Principal Components

An often-used approach when dealing with correlated regressors is principal components analysis or PCA which, when applied to a set of regressors, produces a set of uncorrelated principal components or PCs, which are then capable of explaining the variation in the dependent variable. PCA is a dimensionality reduction algorithm where the first few PCs can capture most of the variation in the data.

Beginning with the set of second-order regressors in Table 12, we implement a PCA algorithm on the data. Figure 13 shows the relationship between the explained variance in the data and the constructed PCs. One can see that the first few PCs capture most of the variation in the data. For example, the first two, three, and four PCs explain 28.7%, 47.7%, and 60.2% of variation that is explained by the 10 regressors. Thus, the 13 regressors can be replaced with a few principal components without much loss in the model's explanatory power.

Figure 14 plots the distribution of the first two and three PCs. The points are colour coded from red (most negative) to yellow (most positive) to illustrate the variation among observations. Note that when abstracting the 10 regressors using two PCs, their distribution on a two-dimensional space is shown on the left panel. Similarly, using three PCs necessitates a three-dimensional space to show their distribution. Since the marginal benefit of using the third PC is only a 12.5 percentage point increase in explanatory power, two PCs can still reasonably capture the variation in the sample.

Although they capture the variation in the data in a parsimonious manner, the problem with including principal components is their lack of intuition as to what each PC depicts or measures. PCs by definition are a linear combination of the other regressors, thus it is possible

Figure 12: Optimal $\alpha$ in Lasso regressions

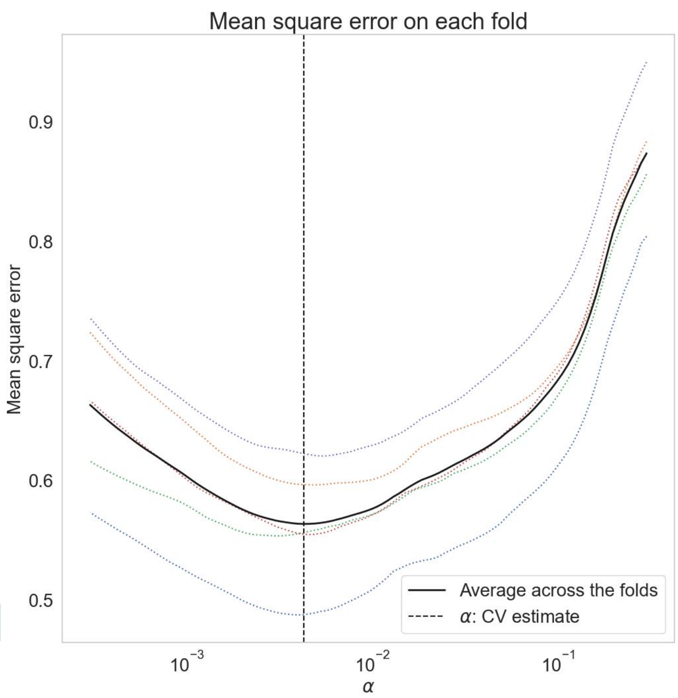



Table 12: Lasso selected regressors

| Weight (importance) | Regressor |
|---|---|
| 0.146 | Deal leverage × Age |
| 0.105 | Size × Profitability |
| 0.089 | Profitability² |
| 0.059 | Industry valuation × Labour intensity |
| 0.047 | Industry valuation² |
| 0.025 | Book leverage × Control |
| 0.025 | Industry valuation × Market price impact |
| 0.022 | Book leverage × Term spread |
| 0.015 | Market share² |
| 0.014 | Market valuation × Labour intensity |
| 0.009 | Book leverage × VIX |
| 0.002 | Industry valuation × Revenue Model Subscription |
| 0.001 | Industry valuation × Sector price impact |

Figure 13: Principal components analysis

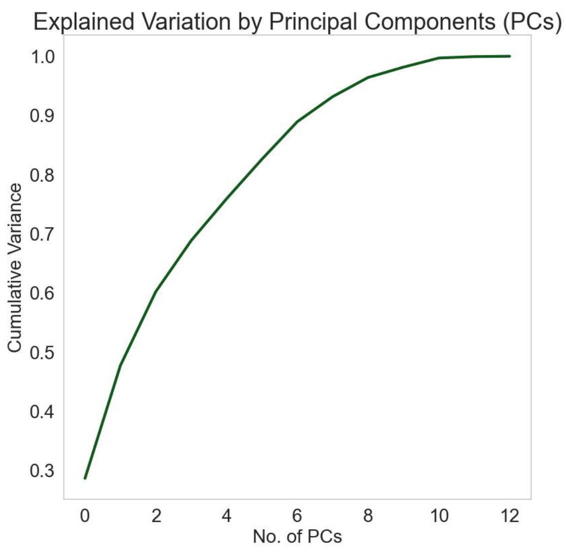
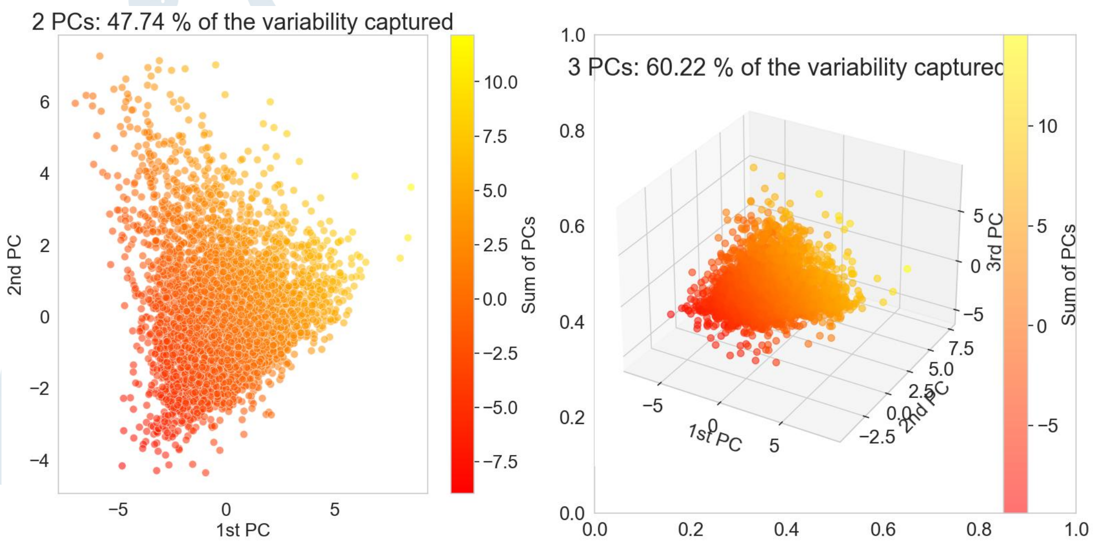



Figure 14: Distribution of principal components




to look at the loadings for the two PCs that are selected and, based on the weights assigned to each of the regressors, they can potentially be renamed on the basis of those regressors that receive the most weight in their constitution. Denoting the first two PCs unimaginatively as PC1 and PC2, Table 13 presents their loading on each of the 13 regressors. Note that the loadings themselves are not comparable to each other as the variables are not standardised and in different scales. However, looking at their loadings or weights gives a sense of which variable affects their composition more. With respect to PC1, leverage measured as book or deal leverage, is more prominently featured, receiving negative weights. Similarly in PC2, **industry valuation** features prominently receiving positive weights. Thus, the first 2 PCs are renamed as **inverse leverage PC** and **industry valuation PC**.

### 6.1.6 Optimal OLS Model

Synthesising the results of all the previous sections, the optimal OLS model is proposed, which comprises of the required variables, forward stepwise regression indicated potential variables, and the principal components obtained from Lasso regression selected second-order variables. Also, in combining

Table 13: Principal component loadings on regressors

| Regressor | PC1 Loading | PC2 Loading |
|---|---|---|
| Deal leverage × Age | -0.317 | -0.102 |
| Size ×Profitability | 0.074 | 0.072 |
| Profitability² | -0.103 | -0.071 |
| Industry valuation × Labour intensity | -0.218 | 0.329 |
| Industry valuation² | -0.215 | 0.485 |
| Book leverage × Control | -0.434 | -0.172 |
| Industry valuation × Market price impact | -0.212 | 0.483 |
| Book leverage × Term spread | -0.448 | -0.223 |
| Market share² | -0.339 | -0.176 |
| Market valuation × Labour intensity | -0.133 | 0.307 |
| Book leverage × VIX | -0.446 | -0.221 |
| Industry valuation × Revenue Model Subscription | -0.103 | 0.237 |
| Industry valuation × Sector price impact | -0.113 | 0.304 |

Variables are standardised after being transformed as indicated in Table 9

these measures, we drop **deal leverage**, **book leverage**, and **industry valuation** from the original model as they are captured prominently in the principal components, and a model that includes them has highly correlated regressors. The results of this model are presented in Table 14.

For the sake of brevity, we do not report the estimates of the PECCS™ indicators, although they are included in the regressions. The interpretation of the coefficients from the OLS model based on the required variables remains unchanged, except **size** which is insignificant. Since size is also included in the principal component computations, the loss of significance in the original **size** variable is less meaningful, provided there is no collinearity problem among the regressors. From the results, one can also observe that all the variables added additionally are significant at least at the 5% level. Specifically, private companies are bought at a higher valuation when they are labour intensive, have patents, belong to hitech sectors, are younger, and are bought during periods with increased liquidity in public equity markets (or lower price impact) and lower value premium in public markets. However, private companies where more control is sought and assets that are purchased as an add-on transaction are valued lower.

The optimal model thus estimated has an adjusted $R^2$ of 0.26. Moreover, the variance inflation factor of the model is a very modest 1.36, indicating no severe multicollinearity problems, despite the inclusion of two principal components. Moreover, in the appendix the pairwise correlations between these optimal variables are reported, which indicates no higher degree of correlation between any pairs.

To summarise the model improvements attained by various econometric methods, Table 15 tabulates the method, the resulting model, and its performance summary. The final model chosen based on the PCA method as being the most optimal (presented in Table 14), which uses lesser interactions, has relatively comparable adjusted $R^2$, and cannot improve performance anymore without adding more variables.

### 6.1.7 Diagnostics of the Optimal OLS Model

Figure 15 plots the histogram of the residuals of the optimal OLS regressions and a scatter plot of predicted versus the actual value of trans-

Table 14: Optimal ordinary least squares regression

| | Dependent variable: P/S | |
|---|---|---|
| Explanatory variables | Estimate ($t$-statistic) | Std Error |
| Intercept | 4.484*** (3.64) | 1.231 |
| Size | -0.069*** (-5.50) | 0.012 |
| Growth | -0.013 (-0.46) | 0.027 |
| Profitability | 0.007*** (17.08) | 0.000 |
| Market valuation | 0.026* (1.83) | 0.014 |
| Term spread | -4.933*** (-2.84) | 1.737 |
| Labour intensity | 0.057*** (6.07) | 0.009 |
| Patent | 0.235*** (8.63) | 0.027 |
| Add on | -0.136*** (-3.94) | 0.034 |
| Hitech | 0.157*** (5.36) | 0.029 |
| Market price impact | -16.680*** (-5.06) | 3.300 |
| Add on | -0.083*** (-4.95) | 0.017 |
| Control | -0.154*** (-3.22) | 0.048 |
| Value factor | -0.002*** (-2.73) | 0.001 |
| Inverse leverage PC | -0.111*** (-10.94) | 0.010 |
| Industry Valuation PC | 0.050*** (4.60) | 0.011 |
| Includes PECCS™ Indicators | Yes | |
| Variance Inflation Factor | 1.365 | |
| Observations | 5,438 | |
| Adjusted $R^2$ | 0.263 | |

Variables are transformed as indicated in Table 9

Table 15: Model summary

| Step | Method | No. of regressors | Adjusted $R^2$ | AIC | BIC |
|---|---|---|---|---|---|
| 1 | Only PECCS™ indicators | 19 | 0.081 | 14,260 | 14,390 |
| 2 | Only use required variables | 26 | 0.205 | 13,470 | 13,650 |
| 3 | Forward stepwise selection | 35 | 0.276 | 12,980 | 13,210 |
| 4 | Polynomial specifications | 5,127 | 0.355 | 12,610 | 14,660 |
| 5 | Lasso feature selection | 39 | 0.289 | 12,880 | 13,140 |
| 6 | Principal Component Analysis (PCA) | 34 | 0.263 | 13,080 | 13,310 |

formed **P/S**. The residuals resemble a Gaussian distribution indicating that the regressions residuals are close to white noise. Moreover, the scatter plot illustrates that most of the points are in the first and third quadrants, which indicates that higher **P/S** is associated with higher predicted **P/S** and vice versa. The dots are also coloured by the PECCS™ activity pillar and there is no discernible pattern among the different activities. The errors of the model are also summarised in Table 16. The mean error in the full sample is almost zero, as the OLS model is supposed to estimate, while the median and absolute error remain small. To check how the model performs out-of-sample, the sample is split into training (80%) and test (20%) datasets, and the coefficients are estimated based on the training sample to predict the values in the test dataset (i.e., out of sample). Such out-of-sample errors are also reported in Table 16. Again, the errors are similar to the in-sample errors and do not raise any significant concerns.

Figure 15: Residuals and scatter plot of optimal model

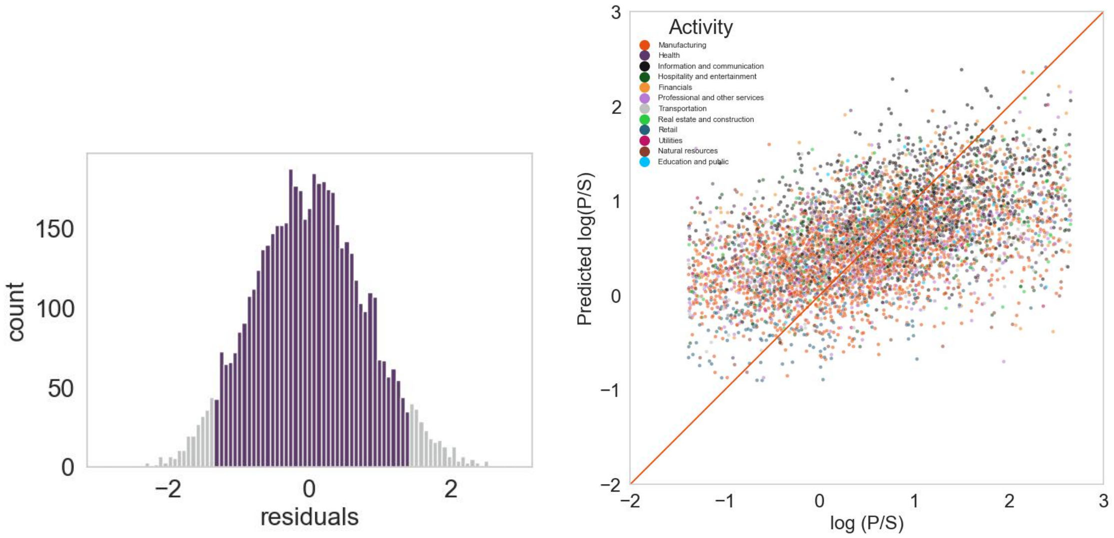



Table 16: Optimal OLS model errors

| Sample | Mean log P/S | Mean error | Median error | Mean abs. error | Median abs. error | Mean sq. error |
|---|---|---|---|---|---|---|
| In-sample | 0.5902 | 0.0000 | -0.0050 | 0.6410 | 0.5434 | 0.6402 |
| Out of sample | 0.5873 | -0.0341 | -0.0650 | 0.6258 | 0.5210 | 0.6020 |

### 6.1.8 Time-varying Effects of Factors

To further examine how severe the problem of time-varying factor prices is, the sample period is divided into 10 ranges of equal number of observations (i.e., 543 or more observations in each period), and the Pearson correlation coefficients between transformed **P/S** and the explanatory variables are computed. These findings are reported in Table 17. Each cell is colour coded ranging from red to green to indicate the minimum and maximum correlations in each column, respectively. Cells with intermediate values of correlation coefficient within each column are coloured in shades between red and green.

Two key observations stand out. First, the length of the periods (i.e., each row) is far from regular, indicating that private company transactions happen in clusters, consistent with Figure 7 showing time trends in deals. Second, the correlations do vary across time. Interestingly, the variation in correlation is not consistent across the columns. For example, **growth** has the highest correlation with log **P/S** in the 2016-2017 period in the sample, but no other explanatory variable reaches such contemporary maximum correlation with **P/S**, except **labour intensity**. Moreover, many variables such as **growth**, **market valuation**, **term spread**, **industry valuation PC**, and **market price impact** change their sign of correlation in certain periods, starkly illustrating the time-varying nature of the relationship between the explanatory and dependent variables.

## 6.2 Dynamic Linear Regressions

To overcome concerns about the static nature of OLS regressions and to account for the time-varying nature of factor prices, a dynamic linear model using the same explanatory variables as in the optimal OLS model is estimated. Specifically, the coefficients are estimated on each transaction date, i.e., when there is new information on a transaction. The filtered coefficient estimates of the dynamic model are estimated which by design captures

Table 17: Time-varying correlation between **P/S** and key explanatory variables

| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1999-2005 | -0.28 | 0.06 | 0.28 | 0.31 | -0.07 | -0.07 | -0.10 | -0.34 | 0.21 | -0.06 | -0.17 | -0.01 |
| 2005-2007 | -0.20 | -0.08 | 0.21 | 0.28 | -0.15 | -0.06 | 0.01 | -0.30 | 0.15 | -0.01 | -0.13 | -0.03 |
| 2007-2010 | -0.29 | 0.00 | 0.25 | 0.20 | 0.04 | -0.11 | -0.10 | -0.34 | 0.16 | -0.04 | -0.22 | -0.07 |
| 2010-2013 | -0.28 | 0.01 | 0.29 | 0.12 | -0.11 | -0.04 | -0.18 | -0.38 | 0.25 | -0.02 | -0.15 | -0.12 |
| 2013-2014 | -0.24 | 0.02 | 0.22 | 0.21 | -0.05 | 0.00 | -0.03 | -0.29 | 0.12 | 0.01 | -0.14 | -0.04 |
| 2014-2016 | -0.28 | 0.04 | 0.28 | 0.27 | -0.16 | 0.02 | -0.06 | -0.37 | 0.13 | 0.02 | -0.27 | -0.12 |
| 2016-2017 | -0.29 | 0.10 | 0.20 | 0.17 | -0.11 | -0.03 | 0.08 | -0.32 | 0.27 | 0.01 | -0.19 | -0.02 |
| 2017-2019 | -0.24 | 0.06 | 0.21 | 0.14 | -0.23 | -0.04 | 0.14 | -0.36 | 0.25 | 0.04 | -0.26 | -0.15 |
| 2019-2020 | -0.23 | 0.07 | 0.23 | 0.23 | -0.22 | 0.01 | 0.13 | -0.36 | 0.15 | -0.06 | -0.24 | -0.16 |
| 2020-2022 | -0.24 | 0.05 | 0.24 | 0.12 | -0.16 | 0.02 | 0.25 | -0.41 | 0.18 | 0.01 | -0.19 | -0.06 |

Colour of cell varies from minimum (red) to maximum (green) in each column
The variables are 1.**Size**, 2.**Growth** , 3.**Book leverage**, 4.**Profitability**, 5.**Market valuation**, 6.**Term spread**, 7.**Industry valuation PC**, 8.**Inverse leverage PC**, 9.**Labour intensity**, 10.**Market price impact**, 11.**Age**, and 12.**Control**.
For the sake of brevity, indicator variables are not presented in the table.

all the information available until the transaction date, akin to how investors in the private companies might incorporate their information sets into prices. For presentation purposes, the smoothed coefficient estimates based on the entire sample are presented, which by design produces smoother trends in prediction by taking into account the entire data during the sample period to smoothen the estimated coefficients. The filtered DLM model results are also quite close to the smoothed results, and for the sake of brevity these are not reported.

Allowing the effect of explanatory variables to be time-varying, relaxes the strict OLS constraint in estimating only the average effect through time. The distributions of the factor prices during the sample period are reported in Table 18. The same evidence is also reported graphically in the appendix, where the average annual factor prices through the sample period are plotted. To show the variability, the 95% confidence intervals of the factor prices are also included, shaded in blue (red) for values above (below) the mean estimate. The effect of each factor in these results is discussed below.

Note that the interpretation of the economic magnitudes of the effects relies on holding other variables constant and whether the predictor is log-transformed or not. If it is not directly log-transformed, the interpretation is obtained by exponentiating the coefficient and subtracting one to obtain the percentage change in **P/S** for a one unit change in the predictor. If it is log-transformed, the coefficient is simply interpreted as the percentage change in **P/S** for a 1% change in the predictor variable.

1. **Size** has a negative effect on firm valuations measured as **P/S**, consistent with the negative and significant effect observed in the initial OLS model that consists of fewer variables. The mean coefficient estimate indicates that a 10% increase in size translates to a decrease in **P/S** of 0.6%. Also, the negative effect of size on valuations has steadily increased over the years during the sample period, indicating a widening small-cap premium on valuation.
2. Although **growth** is expected to increase private company valuation, there is no unconditional support for this notion in the sample, on average, with some periods being characterised by a negative effect, while the later years in the sample period, show a strong positive effect. Theory suggests that value firms may be riskier during downturns (making them cheaper) and that investors may make systematic pricing errors when evaluating earnings. The later years, such as after 2010, do indicate that investors paid higher premiums to acquire private companies that experienced a higher growth. The time-varying effect is also in stark contrast to the OLS results that were insignificant, thus highlighting the power of

using dynamic linear models to perform the regressions.

3. **Profitability** is expected to be positively associated with valuation, supported by the data. The findings indicate that, holding other variables constant, a one unit increase in profitability factor increases the **P/S** ratio by 1.0% ($e^{0.010} - 1$ ). The effect of profitability on valuation also remains positive during the sample period, but its magnitude fluctuates through the years between 0 and 0.03 approximately.
4. **Inverse leverage PC** has a negative effect on valuations of private companies, which is consistent with the positive association observed in the initial OLS regressions that included **book leverage** in Table 11. The **Inverse leverage PC** has higher negative loadings on both book and deal leverage and has a negative and significant effect on valuation, indicating that book and deal leverage, through the first PC has a positive effect on valuation. However, note that this PC has several other loadings on other variables, and its interpretation, does not only rely on leverage changes. Focusing, only on the effect of this variable, a one unit increase in **Inverse leverage PC** decreases **P/S** ratio by 9.5% ($e^{-0.100} - 1$ ).
   The positive effect of leverage measures through the principal components (and also in the OLS regressions in Table 11) may be interpreted as investors assigning higher valuation to companies that are able to pass credit evaluation and subsequently service higher levels of debt. Moreover, incurring higher deal leverage during a transaction signals the confidence of the acquirer or the superior future prospects of the asset.
   Furthermore, the sample period coincides largely with a benign credit cycle characterised by low defaults (with the exception of the subprime mortgage crisis), low interest rates, and higher liquidity in credit markets (e.g., the highly tradable leverage loan markets), which is consistent with higher levels of credit risk tolerance and increased supply of risky debt, thereby allowing the sample to support a positive effect of leverage on private firm valuations. If, on the other hand, defaults were widespread and highly leveraged firms were more subject to them consistently, then it would not be surprising to observe a negative relationship. Thus, the use of dynamic linear models is advantageous in uncovering such evolving relationships, if and when they arise. And as predicted, it is observed that the negative coefficient of **Inverse leverage PC** increases during years of credit boom such as 2000 till 2006 and between 2016 and 2020, whereas in the other years it changes direction.
5. Market conditions play a significant role in affecting the valuation of private companies. The model includes a few measures that can capture market conditions. First, the **market valuation** factor is negatively associated with valuation, in contrast to earlier findings from the OLS model, where a weak positive effect was observed. Holding other variables constant, a one unit increase in market valuation leads to a decrease of 5.6% in **P/S** ($e^{-0.058} - 1$ ). As described earlier, the variable **market valuation** is orthogonalised to the activity pillar of the private company. Therefore, the interpretation is that general public market valuation excluding the industrial activity in which the company operates, has a negative effect on private company valuation.
   Second, **term spread**, measured as the logarithm of a constant plus the difference between long-dated and short-dated government instruments, is found to be positively associated with valuation. A ten percentage point change in **term spread** plus constant increases the **P/S** ratio by 0.1%. Term spread, when zero or negative, can be interpreted as a predictor of future financial crises (e.g., Estrella and Mishkin,

1996), although not employed widely by market participants (e.g., Rudebusch and Williams, 2008). The results are in line with such an interpretation, that when term spread is lower, private equity investors are usually underpaying for transactions in a rational manner.

Third, to accommodate correlated valuations between public and private companies, the **industry valuation** variable is included in the regressions. This measure is positively and significantly related to valuation, as one would expect, observed in Table 11. A similar effect is also observed in the **industry valuation PC** with high loadings on **industry valuation**. Again as the second principal component has multiple loadings on other variables, it is difficult to directly attribute all of the positive effects to industry valuation alone.

Also, liquidity in public equity markets, measured as price impact, is positively associated with private company valuation, in contrast to OLS results. This implies that the more liquid public markets are, characterised by lower levels of price impact, the higher the valuation of private companies, with the effect becoming stronger through the sample period. Finally, the **value factor**, i.e., the difference in returns of value over growth stocks in public markets, is negatively associated on average with private market valuation. In other words, when growth stocks are overpriced in public markets, such pricing errors transcend to private markets as well. However, the effect is highly dependent on time, and the effect keeps changing regularly during our sample period.

6. **Labour intensity** increases the valuation of private companies, with companies with a 10% increase in labour intensity experiencing an increase of 0.7% in the **P/S** ratio. These findings reflect the skills of private equity investors in managing/downsizing large and complex firms through operational engineering, a key pillar of value creation of PE.
7. **Age** of the private firm has a negative effect on its valuation, suggesting that younger private companies are highly valued and earn lower returns. A 10% increase in the age of the private company decreases valuation by 0.8%. Although these results are against the conventional view that younger firms maybe riskier and hence should be undervalued, the findings mirror the observation made in public markets that younger firms may attract overconfident investors who are undeterred by the information uncertainty and still bid higher. Likewise, in private markets where arbitrage opportunities are rare, younger firms seem to be consistently priced higher.
8. In terms of deal characteristics, results show that the percentage of the company sought in the transaction, i.e., **control** and **addon** type of transactions, are negatively related to valuation, decreasing **P/S** measure by 11.9% and 10.3%, respectively, for a unit change in **control** and for add-on type transaction. These findings indicate that investors require higher returns when exercising more control, and thus are willing to pay lower amounts. Similarly, add-on kind of deals are perceived as riskier, because their success depends on realising synergies with the portfolio acquirer company. Therefore, addon companies require higher returns, and thus obtain lower valuations. These effects have been getting more pronounced during the sample period.
9. The technology of a private company also affects its valuation, as demonstrated by the significant coefficient on **Patent** and **Hitech**. **Patent**, measured as an indicator variable that takes the value one when the private company has one or more accepted or pending patents, has a positive effect on valuation. Similarly, a private company categorised as **hitech** commands a higher

valuation. Holding other variables constant, having a patent or operating in a hi-tech segment is associated with an increase in **P/S** of 23.1% and 14.1%, respectively. Moreover the effects of both these measures are seen to be increasing during the sample, reaching a maximum of 2016 and 2018, respectively, for **Patent** and **Hitech**.

10. In terms of PECCS™ classes, results show that private companies in **hospitality and entertainment**, **manufacturing**, **professional services**, **retail**, **transportation**, and **utilities** receive relatively lower valuation while all other activity classes receive higher valuations. All the other pillars of PECCS™ are associated positively with valuation but to varying degrees (i.e., different magnitudes) and in time-varying manner, consistent with the view that investor preferences for private company classes are likely time-varying.

    In terms of non-activity classes, companies that are in the **startup** lifecycle phase, operate **advertising** and **subscription** revenue models, are **consumer focused**, and operate **hybrid** value chains are more valuable than those that belong to other classes, with the effect also time-varying. Comparing the mean coefficients of PECCS™ classes to their standard deviation, the variance is higher than is observed for individual company, deal, or market characteristics.

Table 18: Descriptive statistics of smoothed factor prices from the dynamic linear model

| Variable | Mean | Median | Min | Max | StdDev |
|---|---|---|---|---|---|
| Intercept | 0.0202 | 0.0217 | 0.0101 | 0.0264 | 0.0038 |
| Size | -0.0623 | -0.0648 | -0.1048 | 0.0101 | 0.0221 |
| Growth | 0.0043 | -0.0003 | -0.0664 | 0.0917 | 0.0514 |
| Profitability | 0.0097 | 0.0094 | -0.0069 | 0.0291 | 0.0060 |
| Market valuation | -0.0576 | -0.0696 | -0.0788 | 0.0101 | 0.0225 |
| Term spread | 0.0128 | 0.0131 | 0.0099 | 0.0154 | 0.0013 |
| InvLeverage PC | -0.1003 | -0.0978 | -0.1473 | 0.0101 | 0.0267 |
| IndustryValuation PC | 0.0313 | 0.0248 | 0.0064 | 0.0805 | 0.0196 |
| Labour intensity | 0.0680 | 0.0602 | 0.0101 | 0.1258 | 0.0263 |
| Patent | 0.2076 | 0.2229 | 0.0101 | 0.2870 | 0.0786 |
| Addon | -0.1082 | -0.1218 | -0.1737 | 0.0101 | 0.0436 |
| Hitech | 0.1315 | 0.1555 | 0.0101 | 0.1867 | 0.0516 |
| Market price impact | 0.1026 | 0.1147 | 0.0101 | 0.1629 | 0.0346 |
| Age | -0.0775 | -0.0797 | -0.1137 | 0.0101 | 0.0238 |
| Control | -0.1262 | -0.1486 | -0.2097 | 0.0101 | 0.0706 |
| Value factor | -0.0030 | -0.0040 | -0.0215 | 0.0186 | 0.0072 |
| Activity Education & public | 0.0650 | 0.0700 | 0.0101 | 0.1049 | 0.0330 |
| Activity Financials | 0.1067 | 0.0934 | 0.0101 | 0.1751 | 0.0416 |
| Activity Health | 0.0787 | 0.0805 | 0.0101 | 0.1196 | 0.0290 |
| Activity Hospitality & ent. | -0.0520 | -0.0661 | -0.0783 | 0.0101 | 0.0291 |
| Activity Information & comm. | 0.0253 | 0.0263 | 0.0059 | 0.0505 | 0.0119 |
| Activity Manufacturing | -0.0386 | -0.0400 | -0.0655 | 0.0101 | 0.0184 |
| Activity Natural resources | 0.1505 | 0.1629 | 0.0101 | 0.2375 | 0.0726 |
| Activity Professional & other services | -0.0500 | -0.0552 | -0.0746 | 0.0101 | 0.0191 |
| Activity Real estate & construction | 0.0707 | 0.0724 | 0.0101 | 0.1096 | 0.0253 |
| Activity Retail | -0.1515 | -0.1687 | -0.1897 | 0.0101 | 0.0438 |
| Activity Transportation | -0.0274 | -0.0239 | -0.0687 | 0.0118 | 0.0217 |
| Activity Utilities | -0.0463 | -0.0517 | -0.0673 | 0.0101 | 0.0202 |
| Lifecycle Mature | -0.0041 | -0.0027 | -0.0284 | 0.0274 | 0.0122 |
| Lifecycle Growth | -0.0114 | -0.0080 | -0.0453 | 0.0101 | 0.0132 |
| Lifecycle Startup | 0.0559 | 0.0614 | 0.0101 | 0.0728 | 0.0132 |
| Revenue Model Production | -0.0342 | -0.0334 | -0.0918 | 0.0228 | 0.0329 |
| Revenue Model Advertising | 0.0234 | 0.0249 | -0.0263 | 0.0524 | 0.0202 |
| Revenue Model Subscription | 0.0979 | 0.1041 | 0.0101 | 0.1482 | 0.0358 |
| Revenue Model Reselling | -0.0367 | -0.0349 | -0.0667 | 0.0101 | 0.0154 |
| Customer Model Consumer focused | 0.0648 | 0.0673 | 0.0101 | 0.0897 | 0.0196 |
| Customer Model Business focused | -0.0345 | -0.0379 | -0.0611 | 0.0101 | 0.0188 |
| Value Chain Products | -0.0778 | -0.0852 | -0.1268 | 0.0101 | 0.0321 |
| Value Chain Services | -0.0284 | -0.0331 | -0.0589 | 0.0241 | 0.0246 |
| Value Chain Hybrid | 0.1466 | 0.1586 | 0.0101 | 0.2125 | 0.0500 |

# 7. Robustness Tests

This chapter looks at the results of robustness tests performed on the factor model and the aggregate and sectoral (i.e., each PECCS™ class) level valuation trends based on the smoothed dynamic linear model constructed earlier. We also contrast these trends with contemporary trends in publicly traded markets, namely the S&P 500 and Russell 3000 indexes, which are popular benchmarks for equity investors. Finally, we show the relevant findings when all market-related factors are excluded in order to assuage any concerns that the proposed model is picking up variations in private company valuations only through the inclusion of such factors.

## 7.1 Robustness Checks

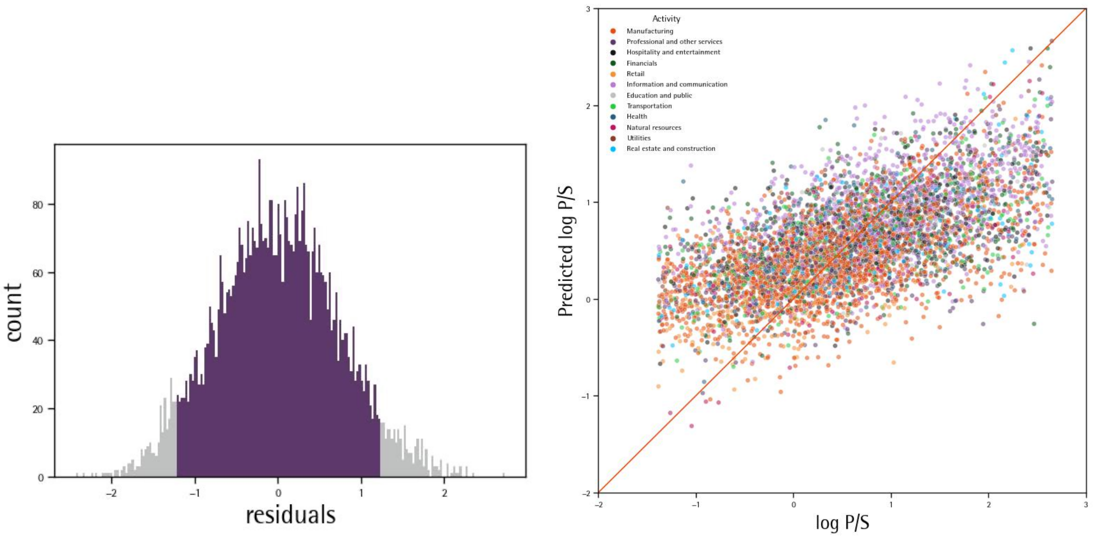
Firstly we look at the results of the robustness tests of the dynamic linear models. After taking into account each factor's individual and independent effect on valuation, the dynamic model requires the idiosyncratic component of valuation to be random, i.e., white noise. To verify this, we plot a histogram of the residuals and a scatter plot of the predicted versus actual values based on the dynamic models. As seen in Figure 16, the residual histogram is similar to a Gaussian distribution, with the exception of a few outliers. Moreover, the scatter plots resemble those of the OLS with the majority of the points in the first and third quadrants.

Additionally, we perform a Box-Ljung test for detecting autocorrelation in the residuals. The *p*-value is lesser than 0.01 for regressions of transformed **P/S**, thus rejecting the null that the residuals are autocorrelated. However, the *p*-value of the Kolmogrov-Smirnov tests for checking the normality of the residuals is less than 0.01, rejecting the null hypothesis that the residuals are normally distributed. Thus, the residuals are reasonably (graphical evidence) but not statistically normal, and are independent and not autocorrelated. This increases the confidence in the coefficient estimates, and they can be considered as being less biased and more robust.

Table 19 shows the errors from the smoothed dynamic linear models. These errors are the difference between the predicted log valuation ratios based on the DLM-smoothed coefficients from the model and the actual observed log value. Although the model predicts the systematic effects of the explanatory variables on valuation, several idiosyncratic characteristics of each transaction in the private market still remain; these are all hard to include in the factor model, both due to a lack of information and due to the difficulty in quantifying idiosyncratic aspects. Thus, the model is expected to predict values that slightly differ from the sample, even if in the sample. Table 19 shows that the average error of the estimates based on the smoothed coefficients is 0.0006 while the median error is -0.0033, which is fairly small and comparable with the OLS models. Moreover, compared to the sample mean of log variable of 0.5900, the errors are very small. The absolute deviations, however, are a bit higher, but the mean and median errors indicate that these absolute deviations cancel out each other across observations.

When viewing the errors at the level of PECCS™ classes, similar patterns emerge, i.e., the mean errors are very close to zero, whereas the median errors remain very small. For example, the highest positive median error of 0.0837 is for companies operating in **subscription** revenue model and the lowest negative median error of -0.1669 is for companies operating in

Figure 16: Residuals and scatter plot of smoothed dynamic linear regressions of **P/S**




the **transportation** sector, which are still small in comparison to sample observations. Thus, even at the PECCS™ classes, the errors are quite tolerable, indicating that the model performs equally well for different PECCS™ classes and for being calibrated predominantly to a few of them. Thus, the factor model is able to capture the systematic effects of on valuation quite well at the aggregate and PECCS™ class levels, even if it does not translate to high accuracy at the transaction level.

Although the modelled variable is closely matched by the predictions, it is also necessary for the raw level of the variable to be accurately predicted. Prior studies have found that transformations can affect the error distributions for the untransformed variable, leading to biased average predictions (Miller, 1984). To ensure that there is no bias due to transformation, we calculate an adjustment factor in the manner prescribed in Cowpertwait and Metcalfe (2009). To confirm that such correction factors work, the mean errors computed as the difference between the original **P/S** ratio and the predicted bias corrected **P/S** ratio are presented in the last column of Table 19. The errors are presented as a percentage of the valuation ratio. Even in the raw measure, the errors are very small.

## 7.2 Sector Valuation

The number of transactions happening in each PECCS™ class or sector is far fewer than would be needed in each period to obtain robust time-series of valuation trends, akin to the problem faced by practitioners when performing comps analysis. Therefore, to examine robust time-series evidence of valuation ratios, the model predicted value of **P/S** is transformed into a moving average of all transactions in the previous 12 months. [1] Using a moving average approach for private company valuation is reasonable as it mimics the comparables approach somewhat where investors rely on recently reported transactions in their comps approach.

Note that when the factor model is applied to a large universe of private companies, it can produce even more robust time series trends in valuations, but such a benchmarking exercise is not the objective of this paper, and is designated as future work.

1 - Since the key dependent variable (i.e., **P/S** ratio) is transformed in the models, it introduces a transformation bias when a predicted transformed **P/S** measure is compared to raw **P/S** as indicated earlier (e.g., Miller, 1984; Cowpertwait and Metcalfe, 2009). To remedy this transformation induced bias in predicted value, a correction factor computed as $exp^{\frac{\sigma^2}{2}}$, where $\sigma^2$ is the cumulative distribution function for a standard normal distribution (CDF) applied on the variance of the sample residuals in the transformed model, is used.

Table 19: DLM in-sample errors

| Sample | Mean log P/S | Mean error | Median error | Mean abs. error | Median abs. error | Mean sq. error | Mean error in P/S |
|---|---|---|---|---|---|---|---|
| Full sample | 0.5900 | 0.0006 | -0.0033 | 0.5881 | 0.4957 | 0.5367 | 1.10% |
| Activity Education & public | 0.7557 | 0.0313 | -0.0459 | 0.5696 | 0.5028 | 0.4901 | 0.84% |
| Activity Financials | 0.8236 | 0.0371 | 0.0821 | 0.6056 | 0.4907 | 0.5749 | 1.82% |
| Activity Health | 0.7375 | 0.0123 | 0.0334 | 0.5968 | 0.5364 | 0.5302 | 2.59% |
| Activity Hospitality & ent. | 0.6689 | -0.0003 | -0.0068 | 0.5693 | 0.4635 | 0.5090 | -1.14% |
| Activity Info. & comm. | 0.9233 | 0.0121 | 0.0671 | 0.5688 | 0.4981 | 0.4986 | -4.44% |
| Activity Manufacturing | 0.4240 | -0.0030 | -0.0122 | 0.5790 | 0.4837 | 0.5226 | 2.47% |
| Activity Natural resources | 0.5705 | 0.0420 | 0.0156 | 0.6371 | 0.5078 | 0.6319 | 9.39% |
| Activity Professional | 0.4765 | -0.0091 | -0.0038 | 0.6017 | 0.5157 | 0.5578 | 3.29% |
| Activity Real estate | 0.6488 | 0.0412 | -0.0256 | 0.6409 | 0.5383 | 0.6555 | 10.96% |
| Activity Retail | 0.0426 | -0.0770 | -0.1068 | 0.5695 | 0.4754 | 0.5166 | 0.53% |
| Activity Transportation | 0.5317 | -0.0186 | -0.1669 | 0.6455 | 0.5619 | 0.6150 | 7.18% |
| Activity Utilities | 0.6287 | -0.0446 | 0.0320 | 0.5717 | 0.4339 | 0.5367 | -8.16% |
| Lifecycle Mature | 0.5056 | 0.0001 | 0.0017 | 0.6020 | 0.5095 | 0.5616 | 2.80% |
| Lifecycle Growth | 0.6940 | -0.0014 | -0.0118 | 0.5610 | 0.4715 | 0.4895 | -1.72% |
| Lifecycle Startup | 0.8752 | 0.0124 | -0.0264 | 0.5778 | 0.4916 | 0.5145 | 0.15% |
| Rev. Model Production | 0.5142 | 0.0000 | -0.0168 | 0.5920 | 0.4978 | 0.5423 | 2.92% |
| Rev. Model Advertising | 0.7237 | 0.0199 | 0.0454 | 0.6017 | 0.5020 | 0.5703 | 1.22% |
| Rev. Model Subscription | 0.9908 | 0.0073 | 0.0837 | 0.5497 | 0.4654 | 0.4713 | -6.89% |
| Rev. Model Reselling | 0.4825 | -0.0114 | -0.0614 | 0.6059 | 0.5091 | 0.5689 | 4.63% |
| Cust. M. Consumer focused | 0.6010 | -0.0011 | -0.0122 | 0.5910 | 0.5007 | 0.5414 | 0.91% |
| Cust. M. Business focused | 0.5707 | 0.0036 | 0.0029 | 0.5829 | 0.4910 | 0.5285 | 1.46% |
| Value Chain Products | 0.6777 | 0.0019 | 0.0043 | 0.5941 | 0.5070 | 0.5476 | 1.13% |
| Value Chain Services | 0.8155 | 0.0355 | 0.0189 | 0.6135 | 0.5169 | 0.5877 | 3.42% |
| Value Chain Hybrid | 0.4689 | -0.0059 | -0.0142 | 0.5783 | 0.4870 | 0.5183 | 0.59% |

*Full names of shortened variables above are available in Table 6.

### 7.2.1 Moving Average Trends

Based on the dynamic linear models, the coefficient estimates of the explanatory variables in the regression are used to obtain a predicted estimate of valuation for each transaction in the sample. For this exercise, the smoothed estimates are used. Results based on the filtered estimates are similar and are not reported. Having obtained the predicted valuations, the moving average is computed of all the firms and at each PECCS™ class level at the end of each month.

The mean **P/S** is computed by applying the correction factor and reverse transformation on $Y$ in the regression, over all transactions ($N$) that have happened in the previous 12 months for each month. This measure is computed for all companies and over each of the classes in the five PECCS™ pillars. This is converted into a monthly panel dataset based on the most recently available average estimate over the sample period.

The findings are presented in the form of graphs in Figures 17 to 22. For comparison, the time-series ratios for the S&P 500 and Russell 3000 indices, popular benchmarks for investors, are also shown. The graph also indicates the correlation coefficients between the private companies' moving average series and the two benchmarks.

Figure 17 shows the aggregate trends. For context on values, the number of transactions happening each month is also included on the secondary y-axis. Further, to illustrate the performance of the dynamic linear model, the first graph in Figure 17 shows the moving average based on the raw transacted valuation of private companies. It can be seen that the raw valuations of private companies are highly correlated with public equities (correlation of at least 0.76), and more volatile than listed stocks.

The second graph in Figure 17 shows the moving average of the predicted values based on the smoothed DLM. Private company valuations remain correlated positively with the public equity benchmarks, with a correlation coefficient of at least 0.79. Because of the smoothing and moving average computations, the aggregate valuation estimates of private companies appear less volatile than those of public equity markets. Moreover, it appears as if private company valuations go up in lockstep with public markets, but fail to correct in a timely manner during declines, consistent with the smoothing proclaimed by private equity fund managers, and surprisingly observable even in transaction prices. Also, notably, the valuations of private companies exceed listed entities for the majority of the sample period, i.e., it seems private equity fund managers are paying more per $ of revenue generated for a private company than investors are paying for publicly listed companies.

Figure 18 breaks down these trends at each activity level. When viewed at the activity level, it can be noticed that at the beginning of the sample, and sometimes throughout the sample period, the trends in valuation are characterised by large swings. This is especially the case when there are fewer transactions in the PECCS™ activity class, as a single transaction can have an outsized effect on the 12-month moving average. From Figure 18, all the sectors for the majority of the sample period are usually valued at a premium over public markets. Such differences can be interpreted in a variety of ways ranging from lifecycle phases, systematic pricing errors, and even a control premium. **Manufacturing** sector displays the strongest correlation of 0.75 with public equities.

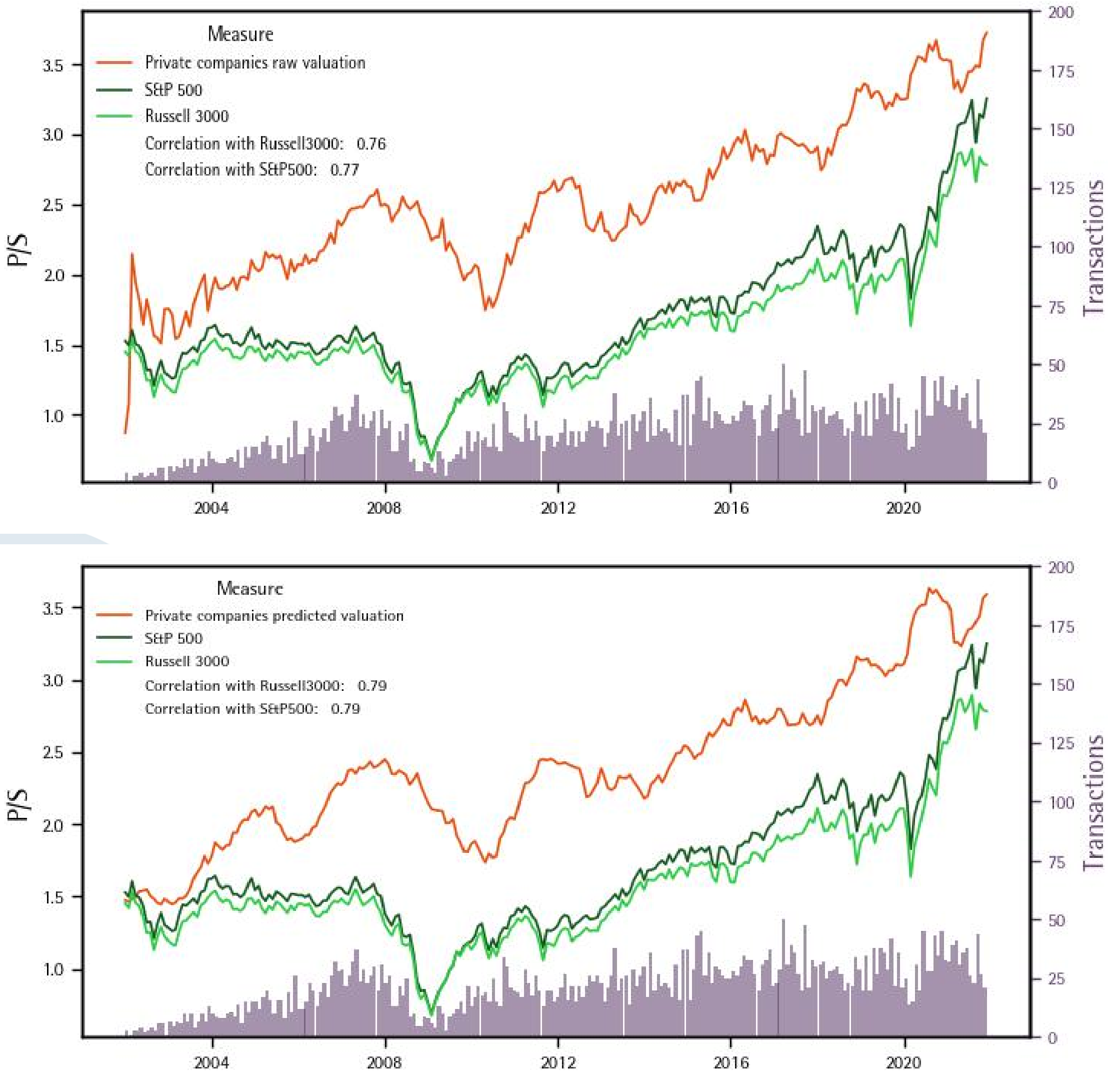
Notably, the valuation of **information and communication** sector has witnessed expanding premium over public markets through the years. **Education and public**, **natural resources**, **real estate and construction**, **transportation**, and **utilities** sectors are characterised by fewer transactions, and hence large swings in valuation along with a low correlation with public markets.

Figure 19 breaks down these trends at each lifecycle phase. Although the majority of the transactions are in **mature** assets, one can find that **startup** companies are consistently valued at a large premium over public equities, are less correlated with public equities, and experience substantial swings in valuations in either direction. **Mature** companies, however, very closely track public valuations, with smaller drawdowns during downturns compared with publicly listed companies.

Figure 20 shows the trends by revenue model. **Subscription** type companies are consistently assigned premium valuations while being moderately correlated with public markets, whereas **advertising** companies, while often above public valuations, experience volatile trends. **Production** and **reselling** model companies closely track trends in public market valuations, with also smaller premiums in valuations compared to **subscription** model.

Figures 21 and 22 display trends by customer model and value chain type. Most classes here follow the stock markets closely, with the exception of **services** type assets which exhibit more volatile trends.

Figure 17: Moving average of predicted valuation of private companies




## 7.3 Excluding Market Factors

The proposed factor model is largely reliant on factors based on the characteristics of private companies. However, the inclusion of some market related factors raises the question of whether all of the variance in valuation is explained by these market based factors such as **market valuation**, **industry valuation**, etc. We can show that this is not the case. First, with the most expansive OLS model that relies on raw factors, we perform an R-squared decomposition to find out the most important ingredients of the final model. Second, we repeat the whole modelling exercise whilst excluding all market related factors. The findings of both these exercises show that the proposed model is not excessively reliant on market related factors, but nonetheless stands to benefit from their inclusion.

The results of the R-squared decomposition are presented in Figure 23, and indicate that the most important variable in explaining the variation in valuation is the **deal leverage**, followed by **labour intensity**, **industry valuation**, **term spread**, and **size**. Thus, it is safe to conclude that even among the top five most important predictors of valuation, only one market based variable is important, i.e., **industry valuation**, and even that is not the most important one in the proposed model.

The results of estimating a dynamic linear model by following all the steps of the previous two chapters and computing moving average

Figure 18: Moving average of predicted valuation of private companies by activity

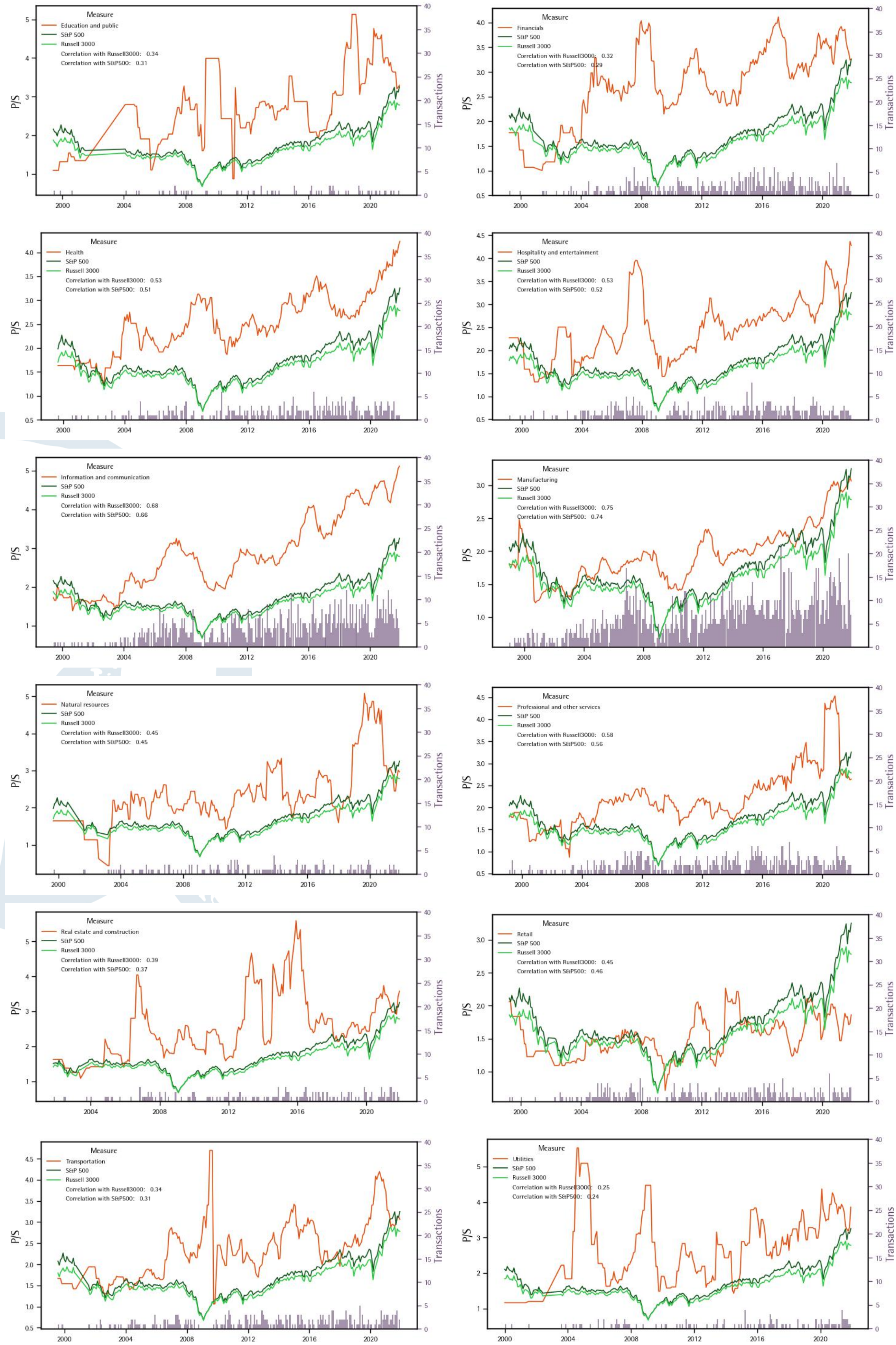


Figure 19: Moving average of predicted valuation of private companies by lifecycle phase

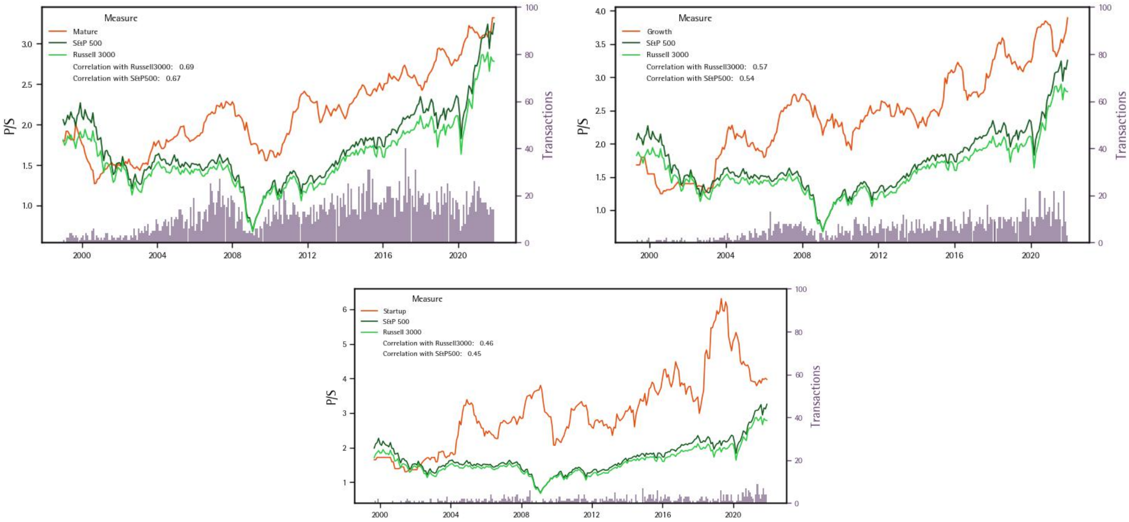



Figure 20: Moving average of predicted valuation of private companies by revenue model

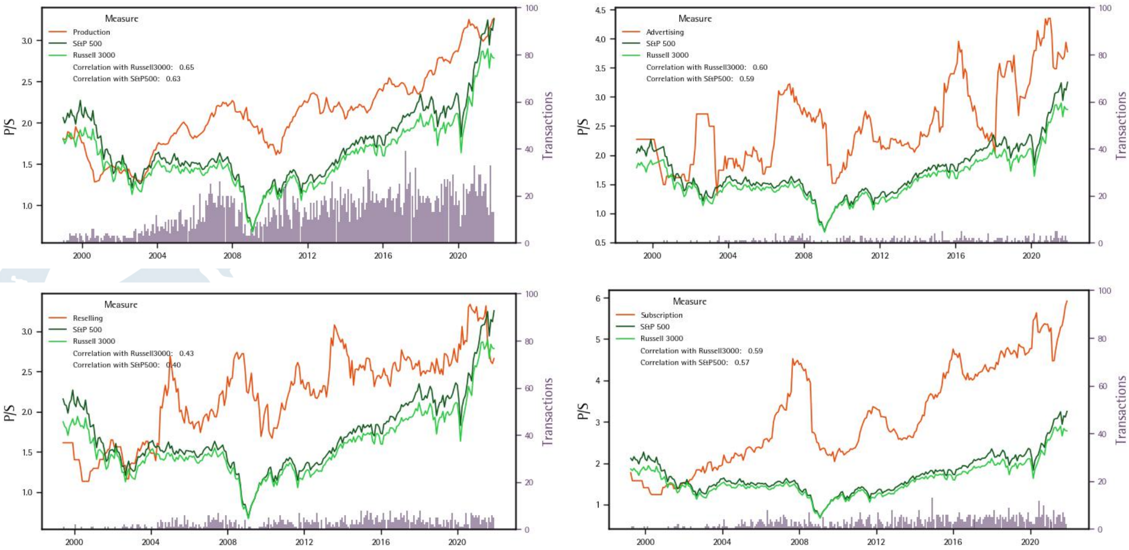



Figure 21: Moving average of predicted valuation of private companies by customer model

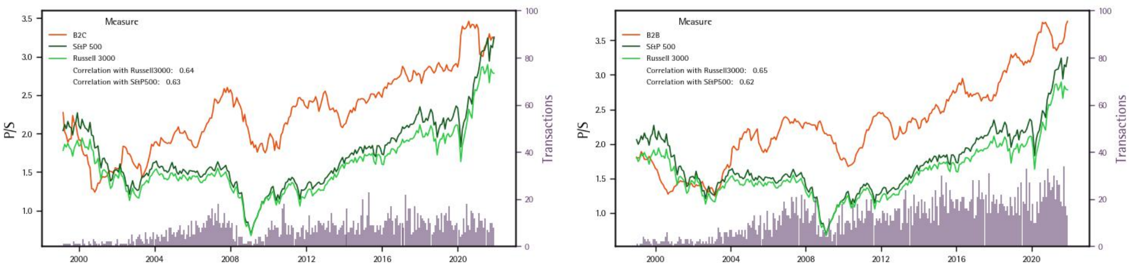
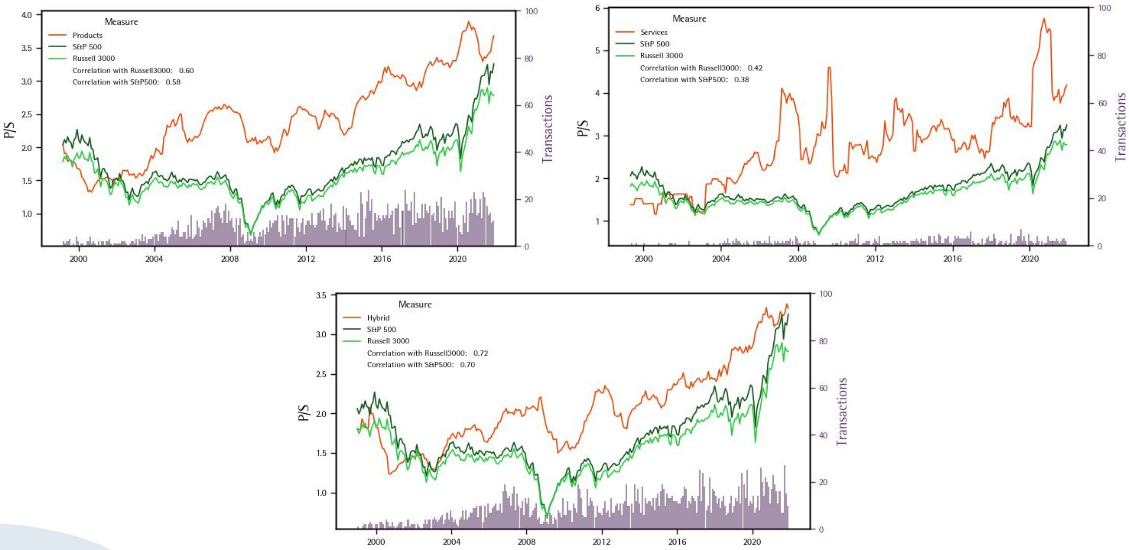


Figure 22: Moving average of predicted valuation of private companies by value chain




time trends in the predicted valuation based on a factor model without market related factors are presented in Figure 24. It is clear from the graph that the predicted valuations based on this alternate factor model are exactly similar to those that include market related factors. Moreover, both series are correlated with a correlation coefficient of 0.999, thus mitigating the concern that market-based factors play a predominant role in the model.

Figure 23: Decomposition of R-square by variables in the OLS model

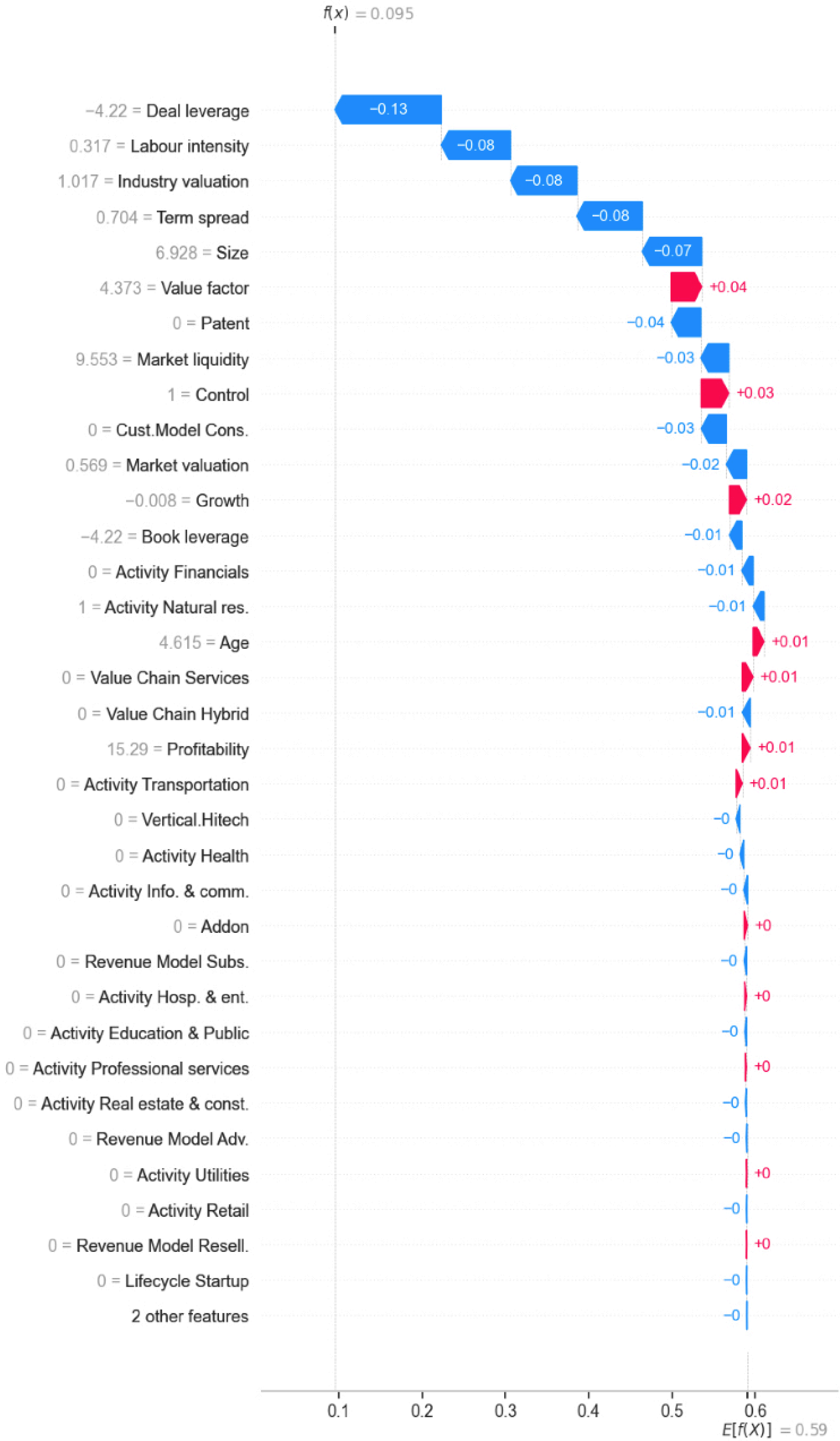
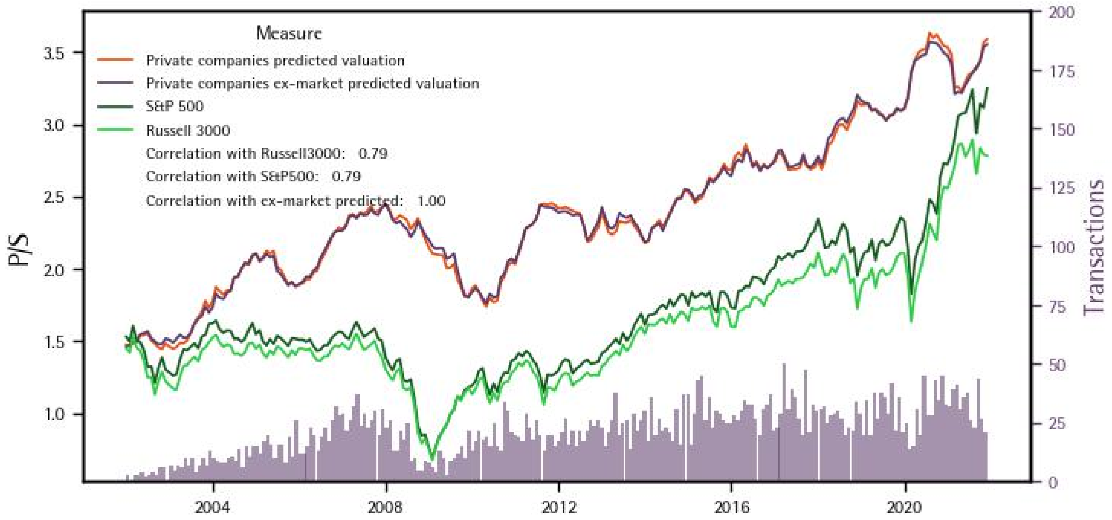


Figure 24: Moving average of predicted valuation of private companies excluding market factors



# 8. Conclusion

Private equity, as an asset class, has grown remarkably in the last few decades (e.g., McKinsey estimates the AUM to be at least $7.62 trillion in 2022). However, there remains great opacity in how these holdings, i.e., private companies are valued. As they are not traded in continuous markets, valuations of private companies are less commonly observable and less comparable. Investors are increasingly unwilling to rely on smoothed, appraised valuations for valuing this important asset class (Braun et al., 2017). Moreover, even when these companies transact, often trading happens in bursts when interest in a specific sector or type of company is higher, thus making raw transaction data a biased proxy of firm value.

In such an environment, creating a representative benchmark for private companies faces several challenges. Current benchmarks rely on GP reported valuations which, even when available at the private company level are very biased, generally being smooth, lagged, and not reflective of all information with the GP. Also, breaking down private company performance from GP fund returns is hindered by the nature of private equity returns, which are made up of a mix of realised cash flows from investments and the appraisal valuations of remaining investments.

Despite the large unaddressed gap that exists in practice, accounting standards emphasise the necessity for fair value standards and require information on recent transactions to be incorporated into the valuation of illiquid assets. Moreover, the democratisation of private investments (and the potential to offer private investments to pension plan participants) increase the demand for accurate and objective valuation of private companies. Thus, there is a demand for data that can address these gaps in private markets, as the production of highly frequent, accurate, and robust valuation data can improve access, reliability, and benchmarking for private market investors.

To address this, this paper proposes a factor model approach, relying on the transaction data of private equity investments in private companies. Our approach overcomes the traditional concerns of using transaction data such as staleness, sparseness, and biasedness; it also avoids relying on the estimated valuations reported by GPs.

Moreover, our approach makes use of a novel private company specific classification standard, PECCS™, which is able to capture the key risk factors from limited information by creating homogeneous groups of private companies across several dimensions, such as their industrial activity, lifecycle phases, revenue models, customer models, and value chain types. The factor model is calibrated using a large, global, and representative sample of private company transactions to reveal how key company, market, and transaction characteristics affect the valuation of private companies.

The factors proposed are grounded in prior academic work, institutional characteristics of private markets, and a survey of GPs. Using econometric techniques to select the best set of factors, our model makes use of dynamic estimation that enables factor prices (i.e., how much premium or discount investors are willing to pay for exposure to a specific factor) to vary with time when investor preferences change. The dynamic linear models allow the determination of latent factor prices based on sparsely observed data that contain noisy information about latent factors. Our factor model is adept at explaining a significant proportion of the variation in the valuation of private companies.

The results indicate that in terms of factors, profitability, leverage, labour intensity, technology, and age of the private company affect its valuation. Additionally, transaction characteristics such as deal leverage, add-on nature, and the percentage of equity sought affect the value. Moreover, contemporaneous market and industry valuations, stock market liquidity, and term spread also affect valuation. Several of these factors also have a time-varying effect on private company valuation. Finally, PECCS™ or PrivatE Company Classification Standard classes of private companies also have a significant time-varying influence on valuation, indicating that such multi-dimensional taxonomies are more suited to private companies.

These results have a broad range of applications:

- **Private company indices**: Using financial information for a large universe of private companies, the factor model can be applied to price non-traded private companies. Pricing a large universe of such assets can enable the construction of indices with different levels of granularity such as a global index, country-level indices, and PECCS™ class levels.
  PECCS™ is a first-principles based taxonomy approach to classify private companies. It helps reinforce the factor model, and in turn, can be validated through it. At the same time, PECCS™ provides a tractable framework to summarise performance and opportunities in private markets.
- **High frequency data**: Indices and benchmarks created using the factor model can be updated frequently, for example at a monthly level, as many of the factor inputs such as market characteristics are available at a monthly frequency. Also, the staggered fiscal years of private companies can generate datasets of private company characteristics that vary throughout the year, allowing the indices to capture such changes. Indices constructed in this way can provide extremely relevant and representative benchmarks to LPs, GPs, and other investors.
- **Custom valuation**: By allowing a standardised approach to estimate valuations, when applied to portfolios of private companies, this approach can remove biases and provide a framework to consistently value portfolios at high frequency. Moreover, applying the models to portfolios can improve their accuracy, as small, estimated valuation errors at the private company level cancel out in the aggregate, producing more reliable return and risk metrics. This can prove beneficial to LPs who carry out valuation tasks over their contributions to several GPs. The framework can also benefit large GPs with multiple funds to manage their individual fund managers.

Moreover, among private companies, granularity and robustness of data are often presented as trade-offs, but the proposed factor model approach, when calibrated with a large dataset of transactions and applied to a large universe of non-traded private companies, can provide highly granular, accurate, and robust segment level metrics.

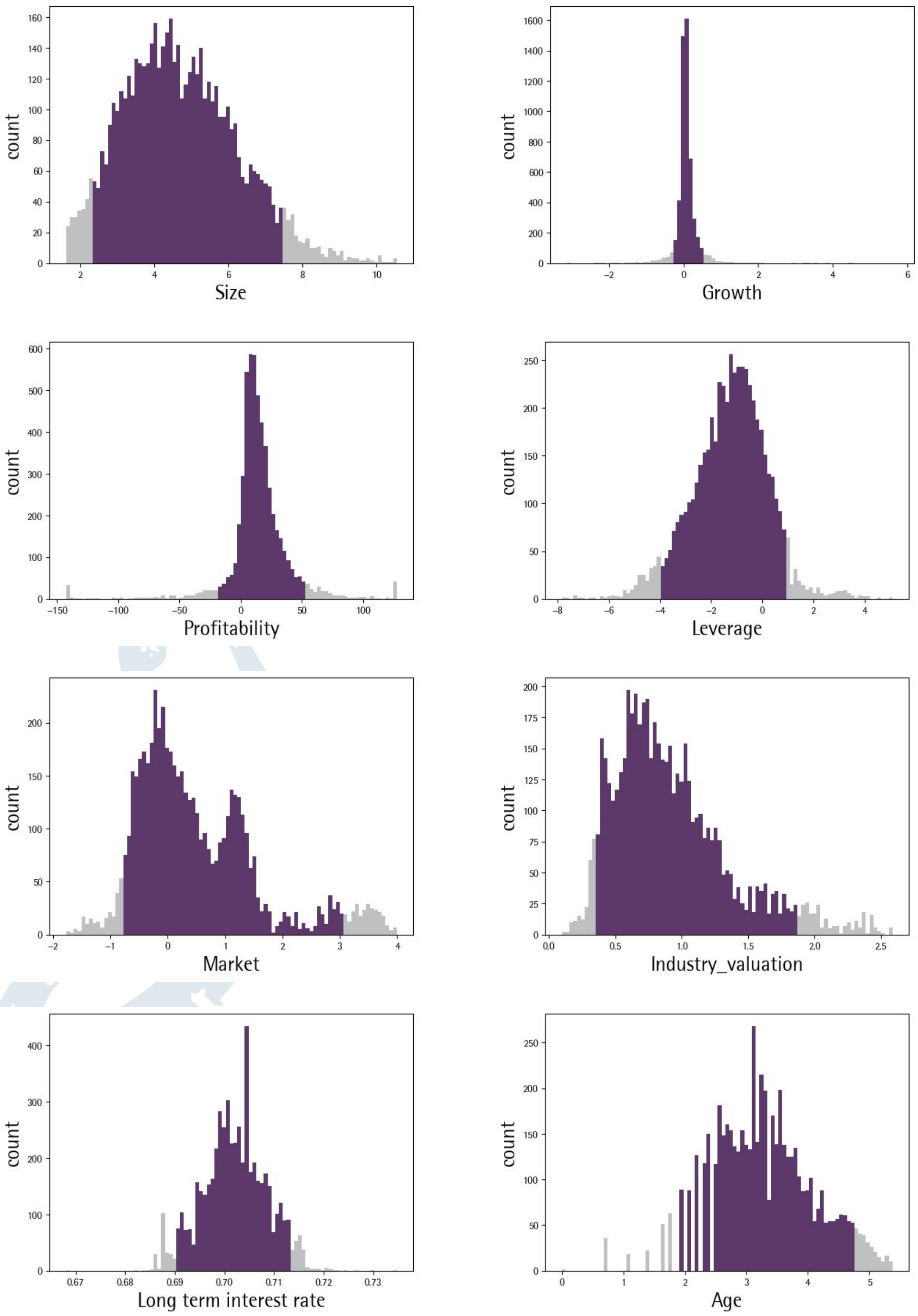
Production of such metrics along with a novel taxonomy of private companies, can facilitate more timely mark-to-market valuation, help overcome the typical biases associated with private company valuation, present a clearer picture of the true diversification benefit of a portfolio of private funds, and ultimately lead to better-informed portfolio allocation and monitoring for investors.

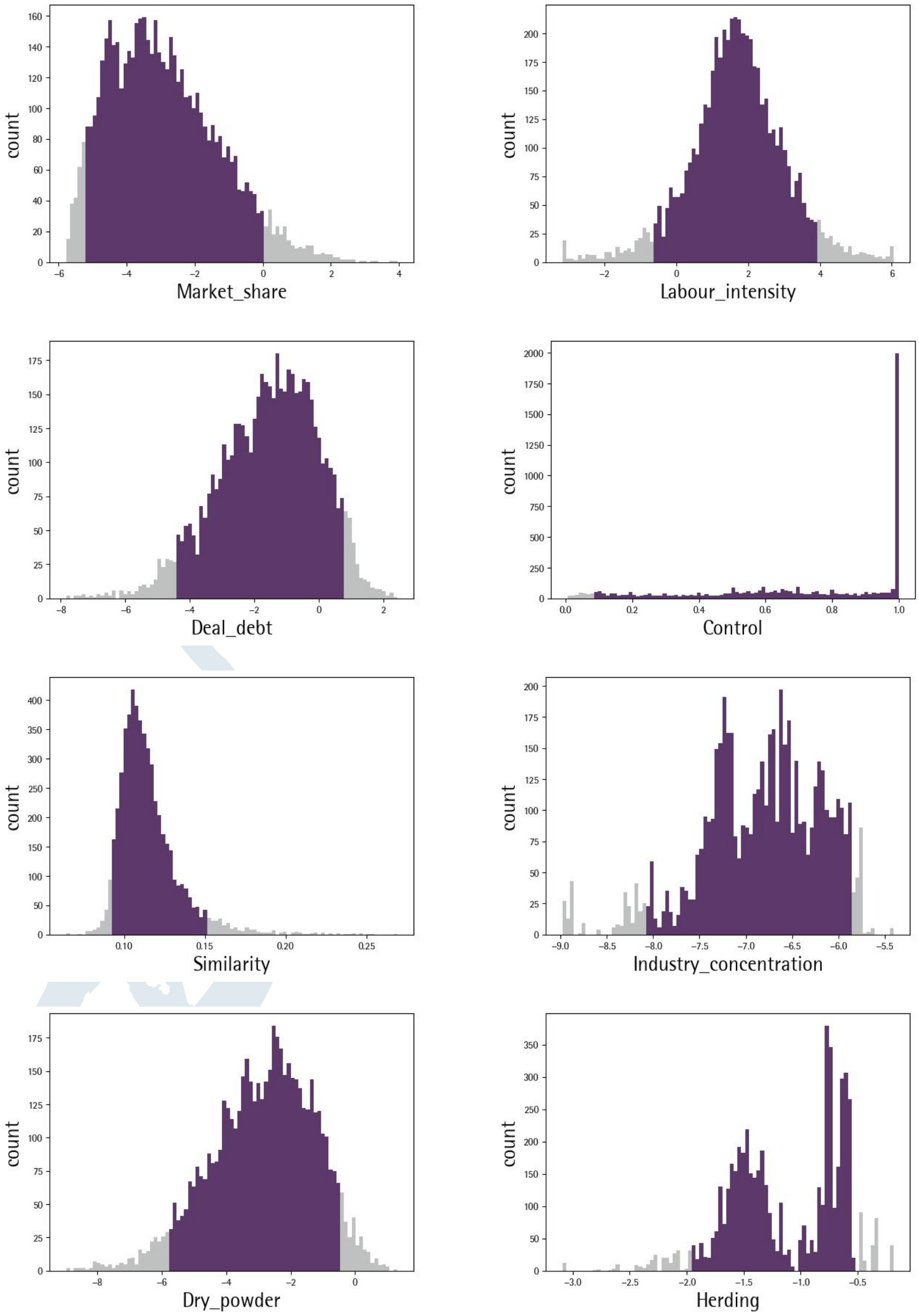
# A. Appendix

The appendix presents supporting evidence in the factor model construction including:

- the plots of the distribution of proposed factors;
- the plots of variation in **P/S** ratios of private companies by key factors; and
- the annual averages of the smoothed factor coefficient estimates through the sample period.

Figure 25: Distributions of Explanatory Variables

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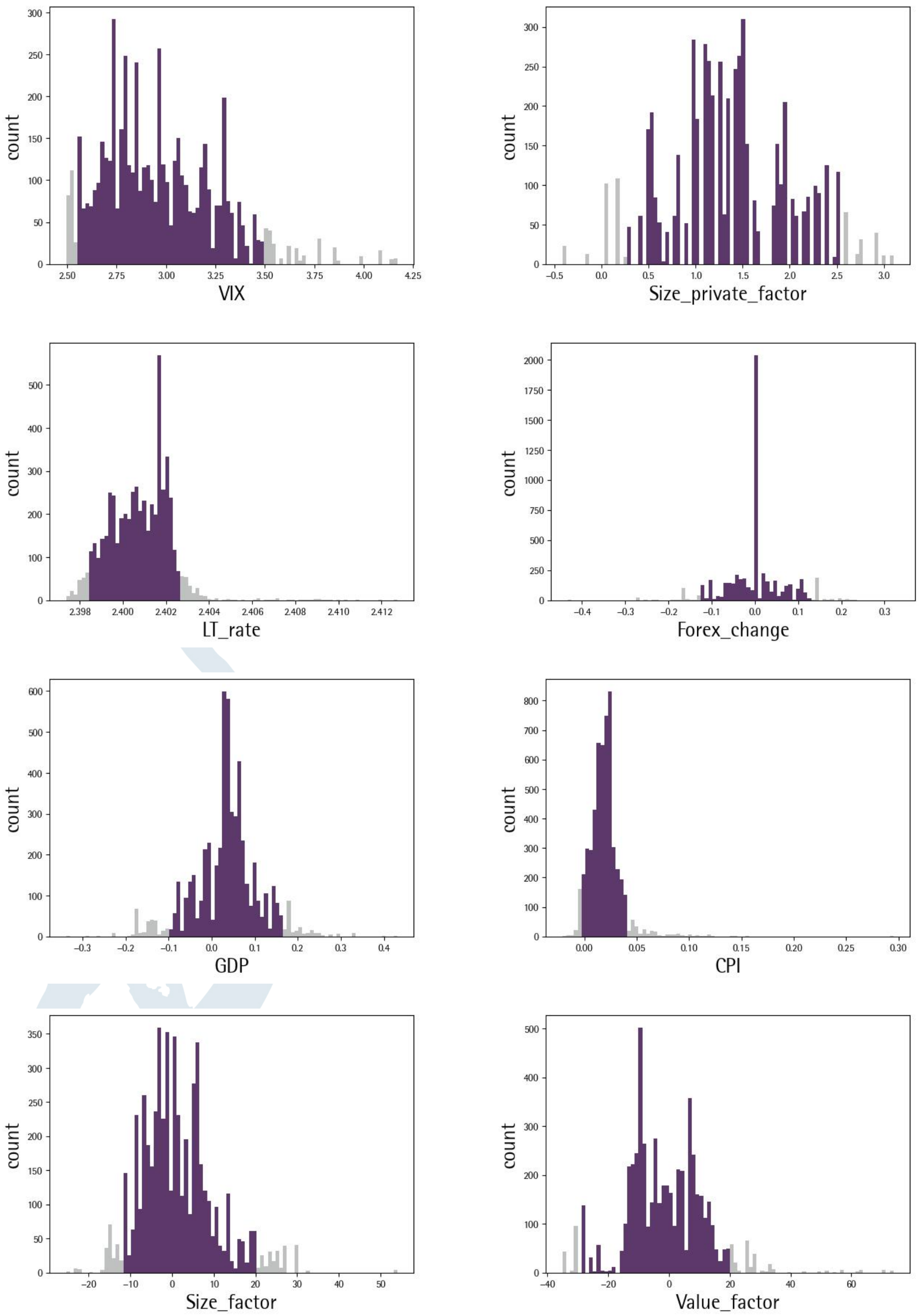
Market_share
count
Labour_intensity
count
Deal_debt
count
Control
count
Similarity
count
Industry_concentration
count
Dry_powder
count
Herding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VIX
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Size_private_factor
count
LT_rate
count
Forex_change
count
GDP
count
CPI
count
Size_factor
count
Value_factor

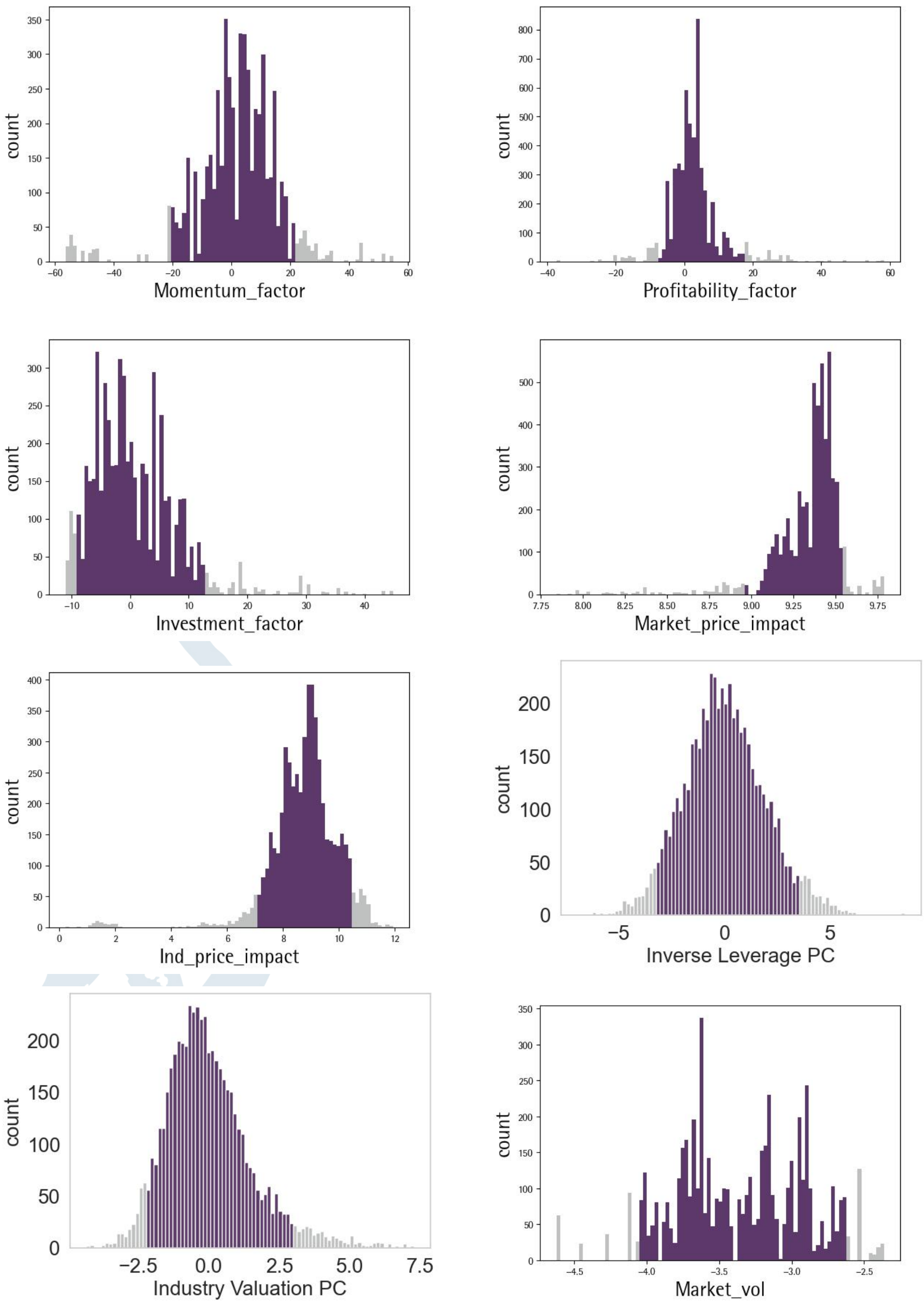

count
Momentum_factor
count
Profitability_factor
count
Investment_factor
count
Market_price_impact
count
Ind_price_impact
count
Inverse Leverage PC
count
Industry Valuation PC
count
Market_vol

Figure 26: Correlation Plots between Key Explanatory Variables

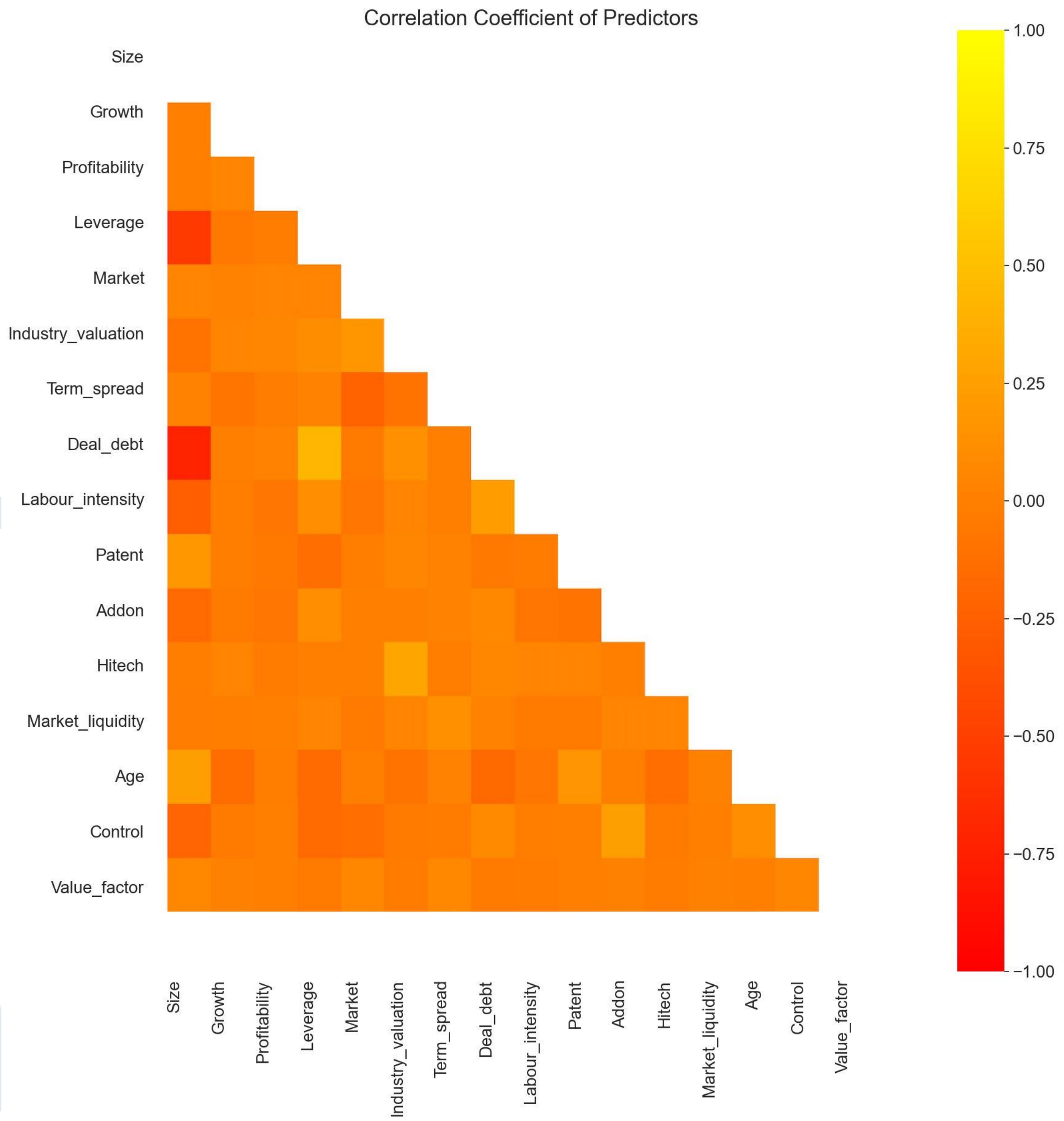


Figure 27: **P/S** ratios by deciles of key firm characteristics

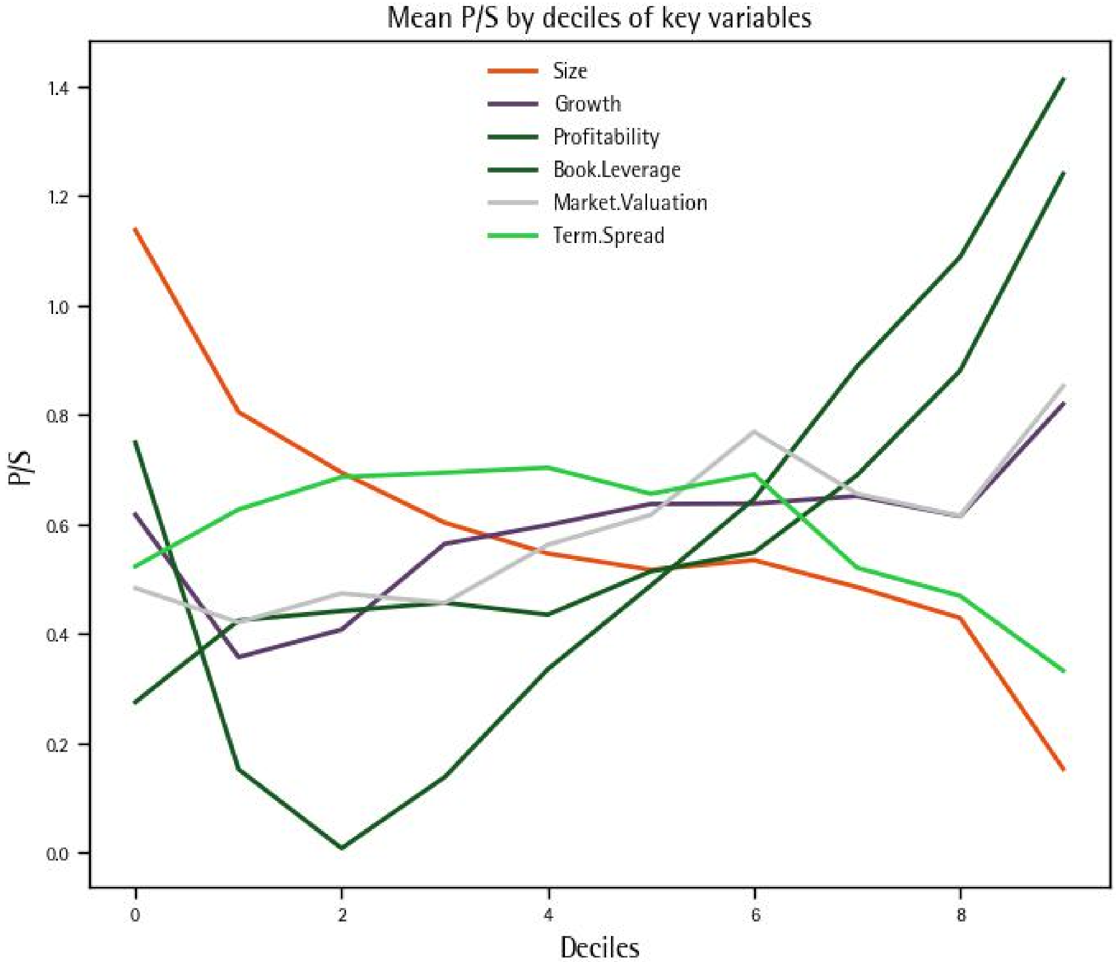


Figure 28: Smoothed time-varying effects of factors on **P/S**

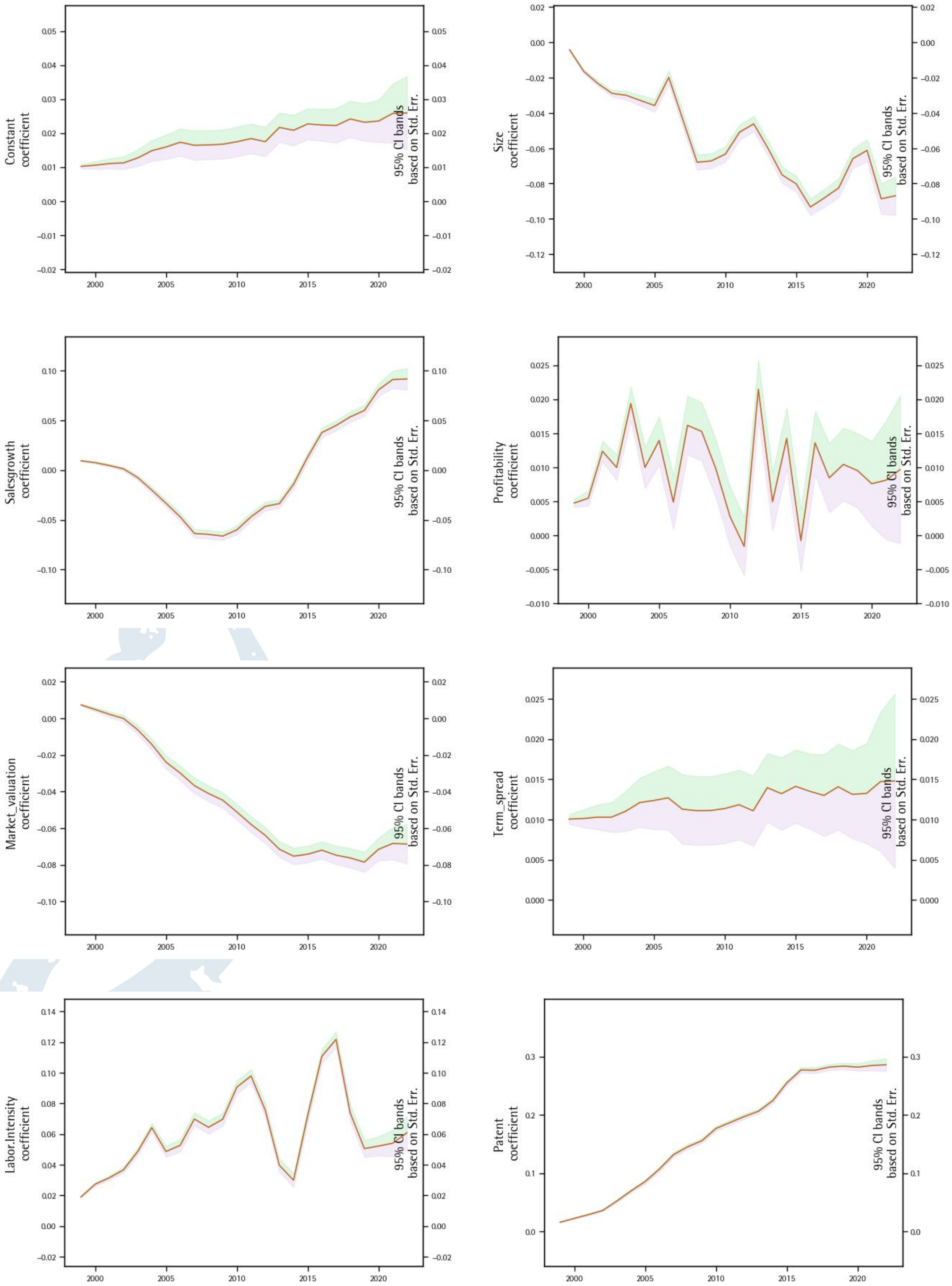


Figure 29: Smoothed time-varying effects of factors on **P/S**

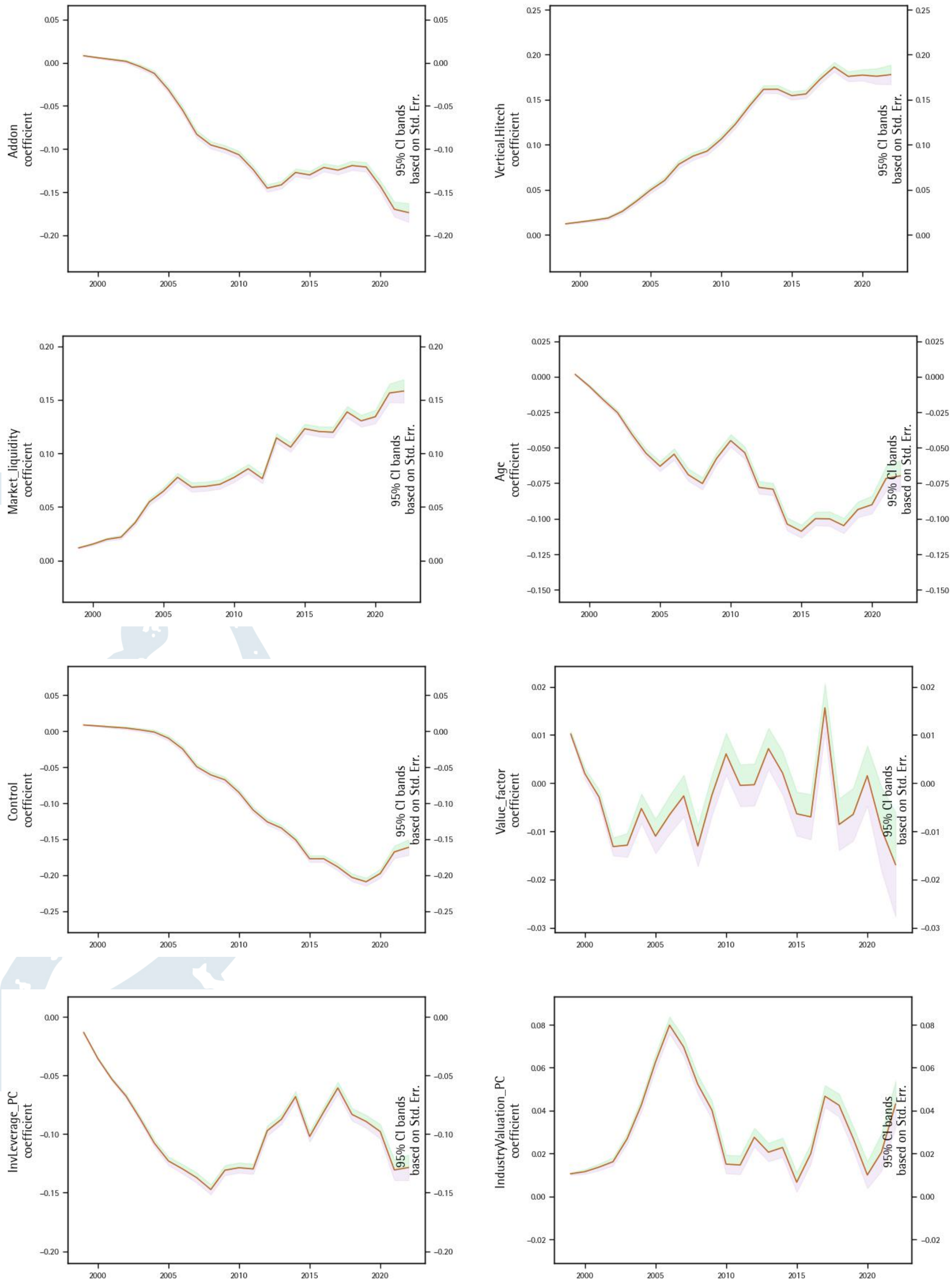


Figure 30: Smoothed time-varying effects of factors on **P/S**

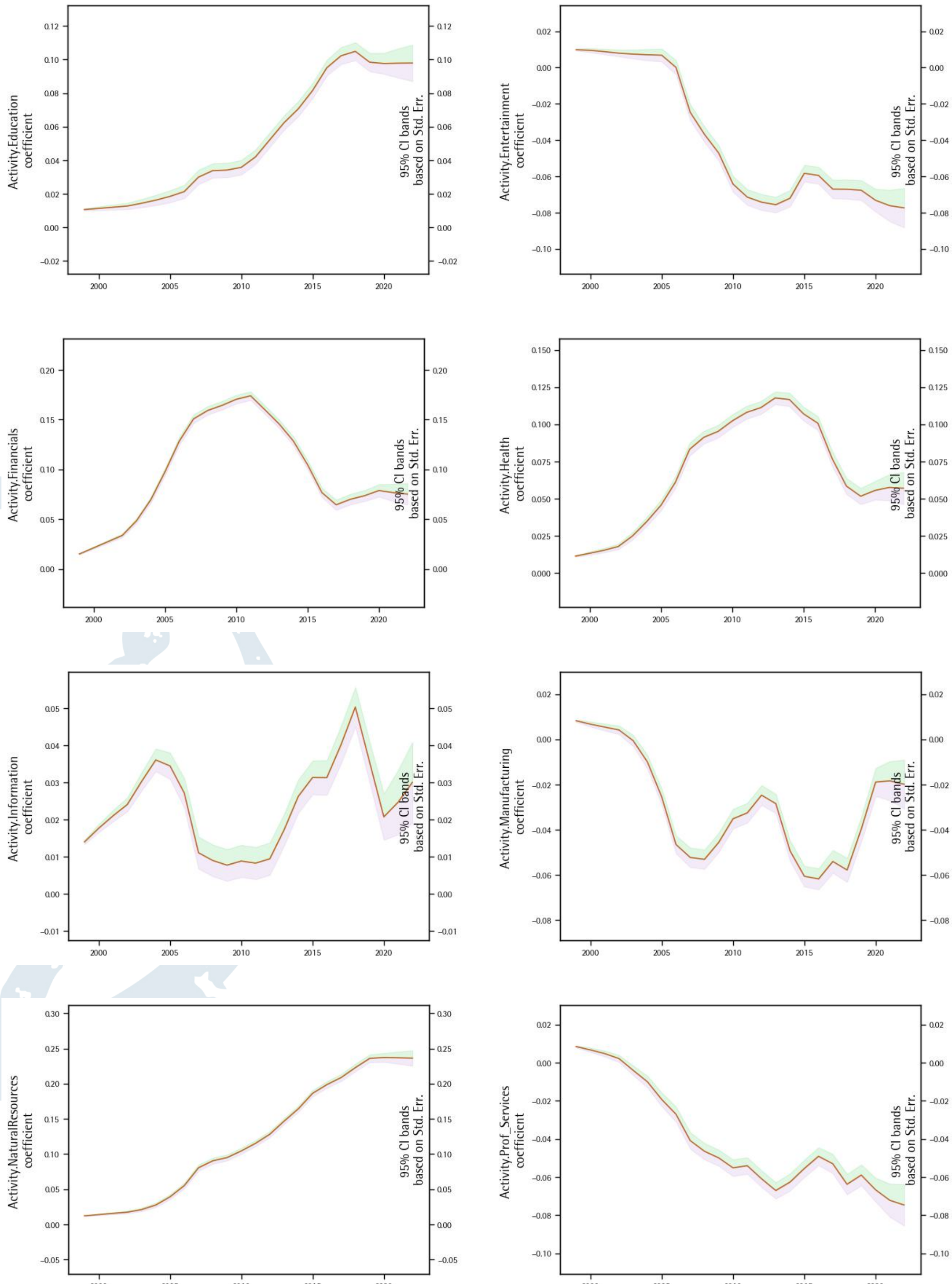


Figure 31: Smoothed time-varying effects of factors on **P/S**

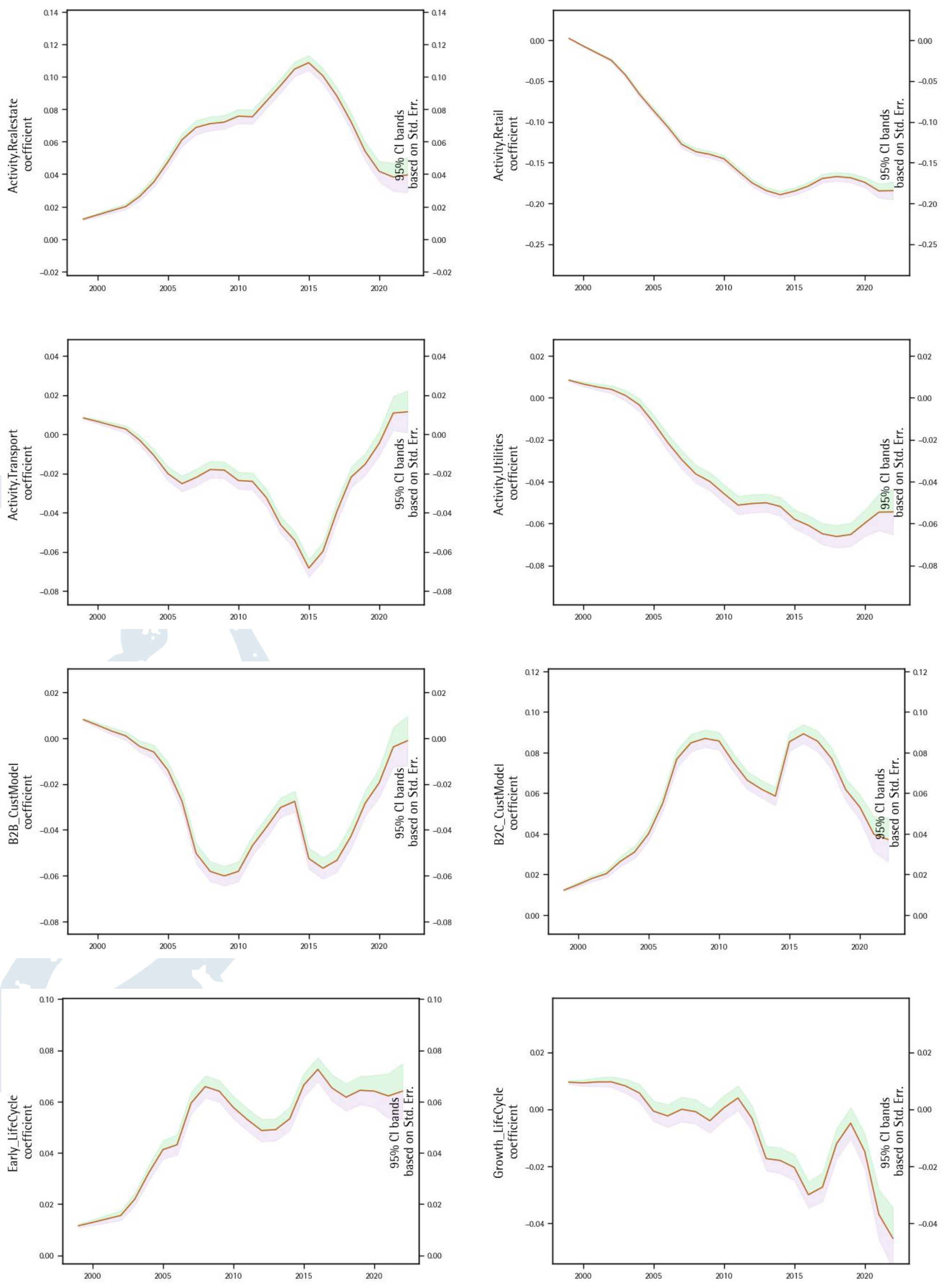
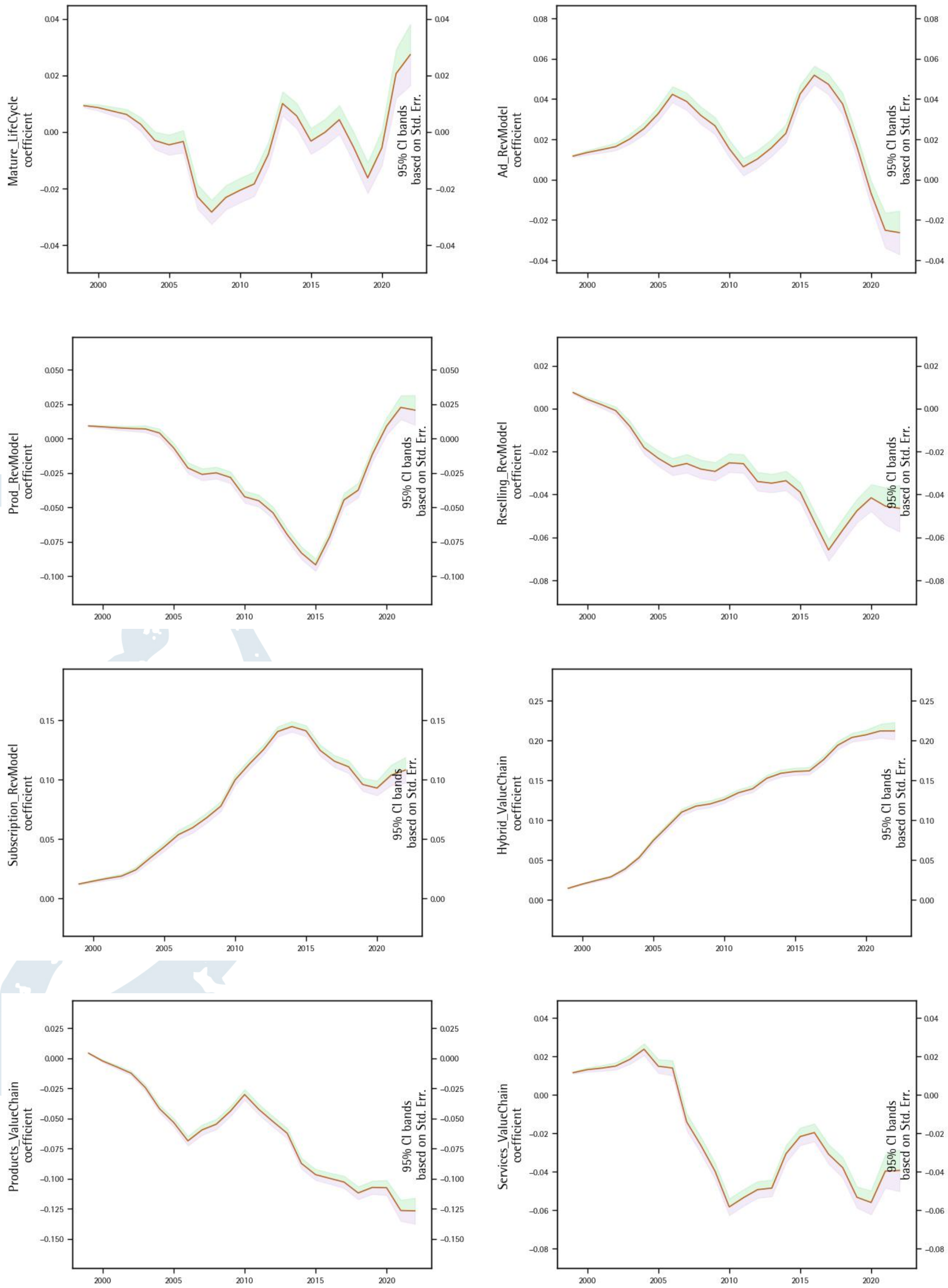


Figure 32: Smoothed time-varying effects of factors on **P/S**



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