You Can Work It Out! *Valuation and Recovery of Private Debt with a Renegotiable Default Threshold*

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This article extends the private debt credit risk model of Blanc-Brude and Hasan [2016] (henceforth, the BBH model) to integrate the role of loan covenants and the embedded options they typically create for creditors in non-recourse financing structures, often referred to as "project financing" (PF). Our extension of the BBH model allows us to compute the expected recovery rates for individuals and portfolios of private loans that take into account the value of embedded options, using a structural credit risk framework requiring a parsimonious set of inputs.

Non-recourse investment projects are typically carried out through a Special Purpose Entity (SPE), using a large proportion of senior unsecured debt. When financing an investment project on a non-recourse basis, senior creditors do not benefit from any collateral to protect themselves in the event of default other than the value of the investment project itself. As a result, they typically impose stringent covenants on the project company (Esty and Megginson [2003], Blanc-Brude and Ismail [2013]) that create significant control rights when covenants are breached.

These covenants can include mandatory reserve accounts for future debt service payments or "lock-ups" barring sponsors from making dividend payouts if free cash flow falls below a certain threshold (Yescombe [2002]). Most notably, credit events create two types of "step-in" options for creditors:

1. An option to require the financial restructuring of the firm in the event of a technical default (a covenant breach that does not lead to a default of payment).
2. An option to take over the entire investment and require the firm's owners to honor their "share pledge" in the event of a default of payment (a.k.a. a *hard* default).

Anecdotal evidence reported by rating agencies suggests that credit instruments originated in the context of project finance transactions exhibit very high recovery rates. For instance, Moody's (Moody's [2015]) reports that a majority of private project finance loans have post-default recovery values in the 80%–100% range and that two-thirds of credit events lead to recoveries of 100%. In contrast, Moody's (Moody's [2009]) reports a long-term historical average recovery rates of 40% for senior unsecured corporate bonds.

The embedded options are not usually found in senior corporate debt, and are thus likely to have a significant value; understanding why creditors achieve such high recovery rates matters to better compare the risk-adjusted performance of non-recourse project debt compared with plain-vanilla corporate debt.

Unfortunately, project finance credit risk analyses are characterized by signifiant data paucity. Recovery evidence is anecdotal because a limited number of defaults are observable, especially after the first decade of loan life, and the number of observed recoveries is even smaller. A number of sampling biases also are likely in existing databases, making the standard frequentist—or reduced form—approach to computing credit risk metrics highly inappropriate.[1]

High recovery rates in PF loans are typically explained in the industry literature by creditors' ability to "work things out" with the borrower after a credit event, as opposed to parting ways, in which case creditors are said to "exit" their relationship with the borrower. Hence, this article proposes an extension of the BBH model of credit risk that integrates the mechanisms by which creditors and project owners choose to "work out" or "exit" a given non-recourse investment project upon a credit event, and what impact their relative bargaining power at that time has on achieved recovery rates.

Our contribution is to provide a technical framework that allows predictions of recovery rates in individual investments for instruments about which there is a dearth of empirical evidence. It is based on a few assumptions of rational decision-making on the part of creditors and borrowers as a function of their relative bargaining power in the case of highly "relationship-specific" investments like infrastructure projects, i.e., investments that have no other use than the one for which they were originally financed.

The rest of this article is structured thus: the first section describes the main characteristics of non-recourse private project debt. The next section briefly discusses approaches to valuing non-recourse private debt and introduces the BBH model, while the following section details our model of the creditor's decision to step in, work out or exit, and the outcome of senior debt restructuring for all parties. The final section proposes an illustrative implementation of the model and discusses some standard results.

NON-RECOURSE PROJECT DEBT

Non-recourse private loans to stand-alone project companies constitute the lions' share of total infrastructure debt (Yescombe [2002]), and also have been well defined since Basel-II: "Project finance is a method of funding in which investors look primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure. In such transactions, investors are usually paid solely or almost exclusively out of the money generated by the contracts for the facility's output, such as the electricity sold by a power plant. The borrower is usually a Special Purpose Entity that is not permitted to perform any function other than developing, owning, and operating the installation. The consequence is that repayment depends primarily on the project's cash flow and on the collateral value of the project's assets." (BIS [2005]).

Most corporations typically hold both tangible and intangible assets and may receive future income from a variety of sources. In contrast, the starting point of non-recourse project financing is that the value of the firm is wholly determined by a well-defined stream of future free cash flows. Hence, firm value is inherently easier to observe and predict: At any time t, it is simply the sum of expected net operating cash flow from the investment project—or *cash flow available for debt service* (CFADS)—discounted at the appropriate rate. This value is the only quantity against which the SPE may initially borrow (or later re-structure or re-finance) any debt.

The only form of collateral available to lenders is the future CFADS including, in particular, the loan's "tail," i.e., the SPE's cash flow available for debt service beyond the original maturity of senior debt, and over which creditors gain control rights in certain states of the world. With this article, our aim is thus to *value the tail* of private project loans.

As argued above, high senior leverage in project finance often warrants strong covenants including "cash sweeps" that distribute excess free cash to debt holders, or minimum DSCR requirements that trigger technical defaults if the DSCR falls below a certain level. Debt covenants also prohibit equity holders from raising more cash through new debt or equity issuance to service existing debt, which directly impacts the default mechanism.

Indeed, as a result, the default point is straightforwardly and uncontroversially known in non-recourse PF compared to standard corporate finance.

As shown in Blanc-Brude and Hasan [2016], the relationship between the firm's CFADS and the expected senior debt service is captured by the debt service cover ratio or (DSCR), which is written:

$$DSCR_t = \frac{CFADS_t}{DS_t^{BC}} \tag{1}$$

in each period $t = 1,2, \ldots, \mathrm{T}$ for a project of maturity T; $\mathrm{DS}^{\mathrm{BC}}$ is the debt service in the "base case," that is, in the current debt contract. Hence, the default threshold is simply defined as $DSCR_t < 1.x$, where $x = 0$ corresponds to a "hard" default of payment, and $x > 0$ corresponds to a "technical" default.[2]

Thus, non-recourse project financing is characterized by a predictable valuation process (the CFADS) and an observable and uncontroversial default point. Moreover, each time the default point is reached, creditors have the option to reset the financial structure of the firm at that point in time, thus **making credit risk endogenously determined.**

Finally, markets for private project debt tend to be both incomplete and not frictionless because of these instruments' illiquidity, lumpiness, and high transaction costs. This is likely to lead to divergent investor valuations determined in part by risk preferences and by the size of the infrastructure debt allocation in their respective portfolios. Hence, a valuation model of unlisted infrastructure loans must incorporate the existence of upper and lower bounds on value, rather than entertain the idea of a unique price.

VALUING ILLIQUID PRIVATE DEBT: THE BBH MODEL

Existing reduced form studies of credit risk in private project debt (such as Moody's [2015]) do not allow computing the evolution of valuation and related risk metrics related to the full distribution of losses, such as expected loss, value-at-risk (VaR), or expected shortfall (conditional VaR).

The role of the covenants and debt restructuring options described above also are not generally incorporated in the existing debt valuation literature, which is typically focused on corporate debt securities. Such covenants and restructuring options are, however, not unique to project finance debt, and can also be found in corporate debt (Chava and Roberts [2008]).

In the academic literature, multinomial tree-based option pricing models have been applied to PF debt (see for example Ho and Liu [2002]; Wibowo [2009]). While these methods can take into account the impact of certain debt covenants, they fail to incorporate the path-dependency and endogenous nature of credit risk in non-recourse project finance.

In the rest of this article, we show that the BBH credit risk framework can be extended to take into account creditors' different options to step in following events of default, the resulting path-dependency of cash flows and credit risk, as well as the heterogeneity of investors' risk preferences due to the illiquid nature of PF loans.

The BBH model (Blanc-Brude and Hasan [2016]) implements a structural approach à la Merton [1974] and consists of the following components:

1. A model of the "state" of the firm: default, no-default, refinancing, lock-up, etc. based on observable DSCR dynamics in the style of Kealhofer [2003]
2. A model of the $CFADS_t$ process and of the distributions to each stakeholders in the various states, given the firm's financial structure and debt covenants, and following its "cash flow waterfall"
3. The risk-neutralization of the cash flow distribution to incorporate a range of risk preferences
4. An adaptation of the Black & Cox model of debt valuation with restructurings (Black and Cox [1976]) to a value process determined by $CFADS_t$ to determine present value of the debt.

Indeed, knowledge of $DSCR_t$ dynamics allows computing the standard distance to default measure and predicting the state of the firm

$$DD_t = \frac{1}{\sigma_{DSCR_t}} \frac{\mathrm{DS}_{t-1}^{\mathrm{BC}}}{\mathrm{DS}_t^{\mathrm{BC}}} \left(1 - \frac{1}{DSCR_t}\right) \tag{2}$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the $DSCR$ value.

The DSCR model leads to a $CFADS_t$ model conditional on the base case debt schedule

$$CFADS_t = DSCR_t \times \mathrm{DS}_t^{\mathrm{BC}} \tag{3}$$

with $\mathrm{DS}_t^{\mathrm{BC}}$, the base case debt service defined at financial close, and the same relationship holds in expectation. This CFADS model can then be used to simulate future cashflows to different stakeholders, while taking into account covenants related to reserve accounts, dividend lockups, and cash sweeps, which are all observable (albeit private) information.

Expected cashflows are discounted to incorporate investors' risk preferences using a risk-neutralized DSCR distribution, thus:

$$F^*(\mathrm{DSCR}_T) = N(N^{-1}[F(\mathrm{DSCR}_T)] + \lambda_T), \qquad (4)$$

where $F(\mathrm{DSCR}_T)$ and $F^*(\mathrm{DSCR}_T)$ are the physical and risk-neutral distributions of DSCR_T.

Finally, extending the Black and Cox model, the value of debt can then be obtained as the probability-weighted expected present value of future cash flows for each possible path taken by the project company

$$V_t^D = \sum_{i=1}^{i=4} h_i(t), \qquad (5)$$

where $h_i(t)$ is the value of the debt at time t from the ith payout function, $V^D(t)$ the total value of debt at time t, and the four payout functions for $i = 1 \ldots 4$ are:

1. $P(T_D, \mathrm{CFADS}_{TD})$: final payment at the maturity of the contract; in the case of non-recourse fully-amortizing project debt this can be set to zero
2. $\underline{P}(\tau, \mathrm{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the lower boundary at time τ, i.e., the default thresholds corresponding to $DSCR = 1.x$
3. $\overline{P}(\tau, \mathrm{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the upper boundary at time τ, i.e., a refinancing threshold
4. $p'(t, \mathrm{CFADS}_t)$: the payments made by the debt security until maturity or the upper or lower thresholds are met.

From here, the value of the firm's senior debt at the upper or lower boundary remains to be determined. The lower boundary is of particular interest since valuation at that point is the expected recovery rate after a default event.

As discussed above, from that point onward, future debt service can be changed following a debt restructuring, also changing the expected DSCR level from that point onwards. To capture this change in the DSCR profile, we need to model the change in debt schedule. Once the change in debt schedule upon default and restructuring can be modeled, we can apply the BBH credit risk model recursively to derive the full distribution of losses in a single project loan or a portfolio of non-recourse project loans.

A RENEGOTIATION MODEL OF NON-RECOURSE PRIVATE LOANS

To model reorganizations, we assume that the equity holders honor their debt obligations as long as there is sufficient CFADS available to make the scheduled debt payment, i.e., there is no so-called strategic debt service (Mella-Barral and Perraudin [1997]), which is a reasonable assumption in PF (see Section 13.5 in Yescombe [2002]).

Next, we differentiate between two types of renegotiations linked to credit events.

Technical Default

We consider technical default events triggered by low realized DSCR levels ($1.x$). Such covenant breaches give debt holders the right to step in and require the restructuring of the outstanding senior debt.

The firm has not filed for bankruptcy and equity holders continue to manage its operations. Neither do they exercise their limited liability option and walk away from their investment.

In this situation, **lenders aim to maximize the value of the restructured debt service relative to the original outstanding debt amount**, which is simply the amortized value of the debt and can be obtained by discounting the remaining scheduled debt payments at the internal rate of return. In other words, their option is limited to maximizing expected recovery up to 100%, but not more.[3]

Debt holders also have to incur *restructuring costs* to have the debt rescheduled. Therefore, they only choose to reschedule the outstanding debt if they can obtain a new debt schedule for which the *risk-adjusted value*, net of restructuring costs, is higher than the risk-adjusted value of the existing debt.

Thus, we simulate non-recourse debt restructuring upon a technical default by implementing the following algorithm:

1. Compute the *outstanding debt value*: the present value of the existing debt schedule discounted at the original IRR of the loan
2. Compute the *risk-adjusted value* of the existing debt schedule, i.e., discounted at the appropriate rate, which is likely to be different from the original IRR, at the time of default

3. Choose a new debt schedule such that its value when discounted at the original IRR of the loan is the same as the original outstanding debt value, and compute its risk-adjusted value
4. If the market value of the new debt schedule, net of rescheduling costs, exceeds the market value of the original debt schedule, the new debt schedule is preferred
5. These steps can be repeated until a debt schedule has been found that maximizes the risk-adjusted value of the restructured debt, for example by minimizing credit risk and extending the debt service in the "tail" of the original loan.

Technical default gives lenders **control rights allowing them to maximize expected recovery.** Technical defaults are the most frequent type of credit event in non-recourse private lending for the obvious reason that the CFADS is more likely to reach a technical default threshold before a hard default occurs.

Hard Default

Hard defaults create a more complex set of outcomes: equity holders lose control of the SPE as a result of their original *share pledge* and creditors have the **option to exit** the relationship with the original borrower and take over the firm and its assets.

However, because of the frequent relationship-specificity of assets financed using non-recourse project financing, *depending on the costs to lenders implied by an actual takeover of the SPE*, the original equity holders have not lost all bargaining power. Hence, **after a hard default, lenders aim to maximize the value of their option to exit** and their preferred course of action may or may not involve the original equity owners.

We assume the following potential outcomes after a hard default:

- Bankruptcy or sale of the company: Debt holders either file for bankruptcy or sell the SPE and receive the salvaged value.
- Takeover: debt holders enter into a new contract with a new set of owners.
- Sale of the loan: debt holders sell the firm's debt in the secondary market.
- Renegotiation: debt and equity holders enter into a new contract.

In the first three cases, debt and equity holders do not renegotiate and we refer to these outcomes jointly as the *exit scenario.* In contrast, in the fourth case—the *work-out scenario*—creditors and the original borrowers agree to continue with the project. Moody's [2015] reports that work-outs are the most frequent outcome following a hard default.

Next, we discuss the conditions under which renegotiations can take place and their possible outcomes.

We denote the value of debt and equity upon liquidation as their *exit value*. Here, debt renegotiation only occurs if the following three conditions are satisfied:

(C_1) Both debt and equity holders can gain at least as much from renegotiation as they can from liquidation, i.e.,

$$V_\tau^i(WK) \geq V_\tau^i(EX), \text{ for } i = D \text{ and } E$$

(C_2) At least one of the stakeholders can get more than what they do under the existing contract, i.e.,

$$V_\tau^i(WK) > \breve{V}_\tau^i, \text{ for } i = D \text{ or } E$$

(C_3) Debt holders never obtain less than the equity holders, as they are the owners, i.e.,

$$V_\tau^D(WK) > V_\tau^E(WK)$$

where D stands for debt, E for equity, WK for work-out, and EX for exit. Thus, $V_\tau^i(WK)$ denotes the value of ith stakeholder ($i \in [D, E]$) upon renegotiation, and $\breve{V}_\tau^i$ denotes the value of ith stakeholder if no change is made to the existing debt schedule.

If the first condition did not hold, at least one of the parties would have no incentive to participate in a renegotiation and it would not occur. If the second condition did not hold, no party would have an incentive to renegotiate. The third condition simply postulates that debt holders, being effective in control of the SPE upon default, should be able to secure at least half of the value of the SPE in a renegotiation.

In the exit scenario (no renegotiation), creditors take over the SPE and either run it themselves or seek new equity investors. Hence the exit value of the debt is the net present value of the cash flows under debt holders' ownership net of any costs associated with taking over the SPE (the exit cost). In this scenario, equity investors are wiped out and their exit value is always zero.

The *exit values* of debt and equity owners (the value of what they receive in the exit scenario) are the lower bound of the *workout values*, and provide an intuitive reason why renegotiation can happen.[4]

Hence, exit values for creditors and equity investors are written:

$$V_\tau^D(EX) = \max(\hat{V}_\tau - X_\tau, \text{Cash}_\tau), \tag{6}$$

$$V_\tau^E(EX) = 0, \tag{7}$$

where $\hat{V}_\tau$ is the exit value of the firm and X_τ represents exit costs at time τ.

We further assume that exit costs are constant in time, workout costs can be either 0 (in the exit scenario) or W (if workout takes place), and debt and equity holders have identical risk preferences and expectations about future cash flows. All of which could be relaxed.

Exhibit 1 shows that if the exit value of the SPE (present value of the cash flows in the exit scenario, net of exit costs) is lower than its workout value (present value of the cash flows under existing ownership net of renegotiation costs), then both debt and equity holders should be better off renegotiating the debt contract, rather than creditors choosing an exit.

In other words, if the workout value of the SPE is sufficiently high compared to its exit value, debt holders can obtain more from a workout, even after sharing a fraction of the SPE with equity owners, who get nothing in the exit case. This is why, despite being notionally wiped out, equity holders still have bargaining power.

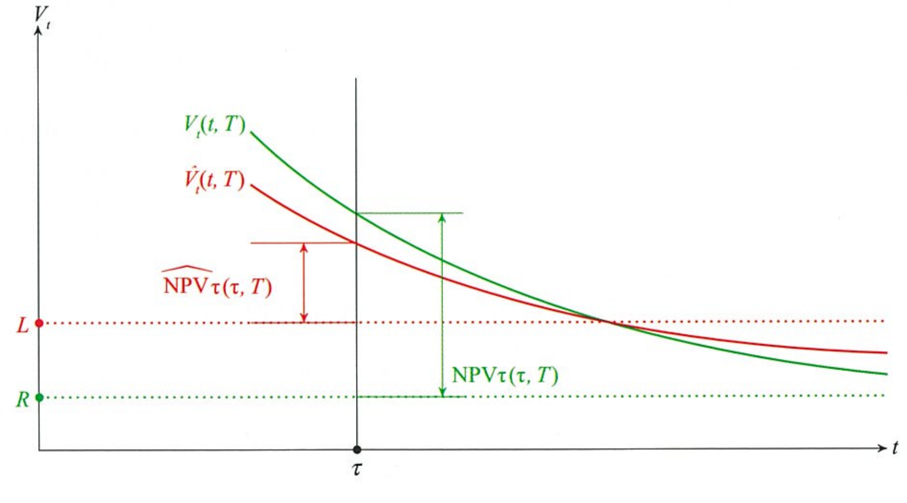
Thus, both the feasibility of a workout and its outcome are influenced primarily by the exit value of the SPE. In the extreme case where the SPE is worth nothing upon exit, debt holders have no choice but to work things out with existing equity holders.

Under this set of assumptions, the value of debt is bounded from below by $max\left((\hat{V}_\tau - X), \frac{1}{2}V_\tau, Cash_\tau\right)$. This is consistent with the commonly used formulas for the outcome of noncooperative bargaining with outside options (Hart and Moore [1994], Osborne and Rubinstein [1990]). Above this lower bound, the following scenarios can be envisaged for the values of debt and equity:

1. $\hat{V}_\tau - X > V_\tau$: The exit value of the firm is greater than its existing value, debt holders are better off liquidating the firm, and there is no attempt to work things out.
2. $max\left(\frac{1}{2}V_\tau, \text{Cash}_\tau\right) < \hat{V}_\tau - X < V_\tau$: The exit value is higher than what debt holders could get by equally sharing the value of the existing firm with the equity owners. This case can be sub-divided into the three possible scenarios:

EXHIBIT 1

Renegotiation and Liquidation Values at the Time of Default



(a) $\hat{V}_\tau - X > \breve{V}_\tau^D$: Debt holders seek to benefit from default, and force equity holders to increase the value of debt to the exit value of the SPV. The new debt and equity values will be:

$$V_\tau^D = \hat{V}_\tau - X, \tag{8}$$

$$V_\tau^E = V_\tau - (\hat{V}_\tau - X). \tag{9}$$

Workout costs will not be incurred in this case, because if equity holders do try to impose renegotiation costs on debt holders, debt holders would simply exit the firm. Hence, equity holders simply let debt holders increase their share of the firm's value, and renegotiation is costless for creditors.

(b) $V_\tau - (\hat{V}_\tau - X) - W > \breve{V}_\tau^E$: Here, equity holders can benefit from default and force debt holders to make concessions and reduce the value of the debt to the exit value of the SPV. The new values of debt and equity will be:

$$V_\tau^D = \hat{V}_\tau - X, \tag{10}$$

$$V_\tau^E = V_\tau - (\hat{V}_\tau - X) - W. \tag{11}$$

In this case, the equity holders would have to incur workout costs, because debt holders do not lower their share unless equity holders force them to do so.

(c) Neither of the above two conditions hold: In that case neither party stands to benefit from default, and they simply continue with the existing debt schedule. Creditors "waive" the event of default.

$$V_\tau^D = \breve{V}_\tau^D, \tag{12}$$

$$V_\tau^E = \breve{V}_\tau^E \tag{13}$$

3. $\frac{1}{2}V_\tau > max(\hat{V}_\tau - X, \text{Cash}_\tau)$: The value of the exit option is so low that debt holders are better off equally sharing the existing value of the firm with equity holders. The new values of debt and equity are given by:

$$V_\tau^D = \frac{1}{2}V_\tau, \tag{14}$$

$$V_\tau^E = \frac{1}{2}V_\tau. \tag{15}$$

4. $\text{Cash}_\tau > max\left(\hat{V}_\tau - X, \frac{1}{2}V_\tau\right)$: In this last possible case, the value of cash available in the current period is greater than the value of SPV as a going concern. Hence, debt holders simply take the cash at bank and the firm ceases operations. In practice, equity holders could offer to allow debt holders to keep the available cash and continue to run the firm. Here, debt holders have no incentive to work things out, and we assume that the firm ceases to exist. The new debt and equity values are:

$$V_\tau^D = \text{Cash}_\tau, \tag{16}$$

$$V_\tau^E = 0. \tag{17}$$

Applying this model, a new debt schedule upon a hard default can be computed such that the present value of the debt schedule is equal to the value of new debt given by the renegotiation model.

In principle, many different debt schedules can be determined that yield the same present value. For simplicity, in the illustration proposed in the next section, we assume that the new debt schedule is determined such that the loan has a constant DSCR, and the maturity of the loan coincides with the maturity of the project.

ILLUSTRATION

Once the outcome of debt restructuring is known, total debt value can be computed following the Black and Cox decomposition given in Equation 5 and iterating through all the possible paths of the firm's cash flows for a given DSCR process.

In this section, we compute risk and return measures for the two generic merchant and contracted infrastructure project debt described in more detail in Blanc-Brude and Hasan [2016].

Assumptions

- Equity dividends are locked up if the DSCR falls below 1.10, and technical default is triggered if DSCR falls below 1.05.
- Exit costs X are 60% of the face value of debt.

- Work-out costs W are one-half of exit costs.
- Restructuring costs are one-third of exit costs.
- We ignore any termination payment due by a "grantor"—typically the public authority that granted the concession to the SPE—that would automatically create a floor for creditors' exit value. Such guarantees exist in a number of projects but are not systematically found in project finance.[5] Such a floor could easily be added as an extension of the model.

DSCR Families

The DSCR process for a typical "merchant" project is modeled using a lognormal distribution with a constant mean return of 1%, a constant volatility of returns of 3%, an initial DSCR of 1.4, and 20% volatility of the initial DSCR.

$$\frac{d(DSCR_t)}{DSCR_t} = \mu dt + \sigma dW_t, \tag{18}$$

The DSCR process for "contracted" projects is modeled using a normal distribution with a mean DSCR of 1.2 and a volatility of 8%.

$$DSCR_t = E[DSCR] + \sigma(DSCR) dW_t. \tag{19}$$

These DSCR profiles are illustrated in Exhibit 2. We note that these assumptions are reasonable and in line with the findings of a recent paper using a new dataset of several hundred non-recourse projects in the OECD (Blanc-Brude, Hasan, and Whittaker [2016]), in which the authors show that the empirical distribution of $DSCR_t$ does follow an auto-regressive lognormal process at least during the first 18 years of project loan life, and that merchant and contracted projects follow statistically different DSCR processes.

Restructuring Simulations

Technical default. We illustrate the outcome of the debt restructuring model with a simple example and consider either a technical default or a hard default at time $t = 10$. As the same procedure is followed for both families, we only discuss the procedure for the rising (merchant) DSCR family.

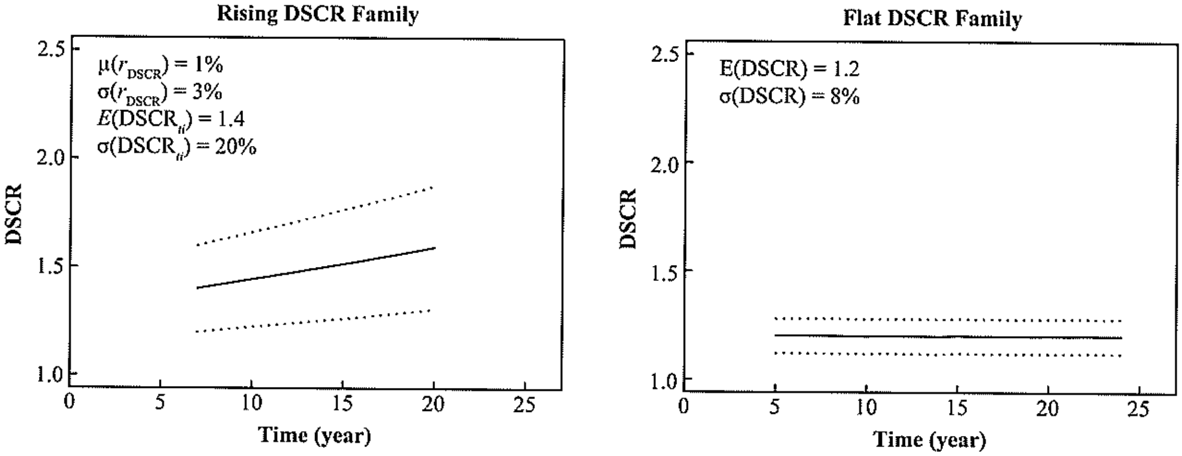
In this run of the simulation, the realized CFADS at $t = 10$ is \$131, barely sufficient to satisfy the scheduled debt payment of \$130.7737 in that period. The SPV is in technical default and creditors can reschedule the outstanding debt. As before, creditors reschedule the outstanding debt if the risk-adjusted value of the *rescheduled debt* (net of rescheduling costs) exceeds the market value of the existing debt schedule.

In this example, the amount of debt outstanding at $t = 10$ is \$1,003.59 (the present value of future debt payments discounted at the initial IRR of the loan). Here, senior debt is rescheduled using a constant amortization profile, but other assumptions could be used depending on the original amortization profile.

Exhibit 3 shows the rescheduled debt service in two separate scenarios: 1) when rescheduling costs are

EXHIBIT 2

Physical and Risk-Neutral DSCR Distribution for Merchant and Contracted Infrastructure Projects


Rising DSCR Family
Flat DSCR Family

high, at \$100, and 2) when rescheduling costs are lower, at \$10. In the first case, high rescheduling costs mean that creditors cannot find a new debt schedule with a constant amortization profile that exceeds the existing debt schedule's market value, and the debt schedule remains unchanged. In the second case, lower rescheduling costs mean that the optimal debt schedule is the one with the longest possible maturity. The new constant debt service is \$93.44.

Note that both debt schedules—the initial one with a constant debt payment of \$130.77 until year 20, and the new one with a constant debt payment of \$93.44 until year $t = 25$—have the same amortized value of \$1003.59 at $t = 10$. Yet, the two debt schedules have different *risk-adjusted values*, which reflects the expected default frequency in each scenario and the exit value, while the amortized value does not. By selecting a lower annual debt service compared to the initial one, creditors have decreased the firm's probability of default and increased the risk-adjusted value of the loan, without changing its amortized value of debt. **This is the value of the step-in option in a case of technical default**.

Next, if the optimal debt schedule is the one with the longest possible maturity, it is because—following the Black & Cox decomposition—the risk-adjusted value of the loan also includes the lenders' exit value. If their exit value is small, which is the case in this simulation since exit costs are high, debt holders are better off minimizing the probability of default even at the expense of higher duration.

Hard default. Next, we examine the simulation of the restructuring upon a hard default, also at $t = 10$. In this run of the model, realized CFADS upon default is \$110. Exhibit 4 shows two ways to reschedule debt upon hard default for an exit value of \$601.16. The red line shows the rescheduled debt when debt holders choose to reschedule debt until the original maturity of the debt (20 years), and the blue line shows the rescheduled debt when debt holders choose to reschedule debt until the maturity of the project (25 years).

The two debt schedules have different maturities, but both have the same risk-adjusted value: the risk-adjusted value must equal creditors' exit value, whichever new debt schedules and new amortized value is preferred.

This is the main difference between debt rescheduling upon technical or hard defaults: after a technical default, the amortized value of the debt remains the same, since creditors can only maximize their recovery rate up

EXHIBIT 3

Debt Rescheduling upon Technical Default, BC = Base Case, C = Rescheduling Cost

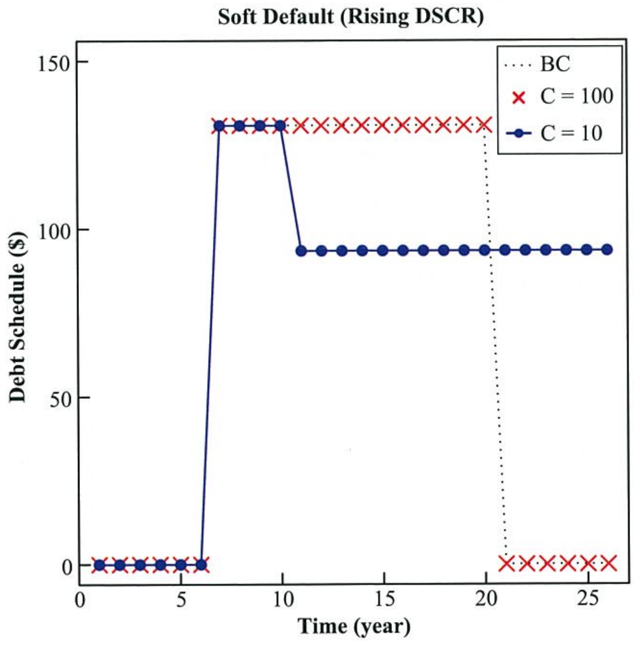
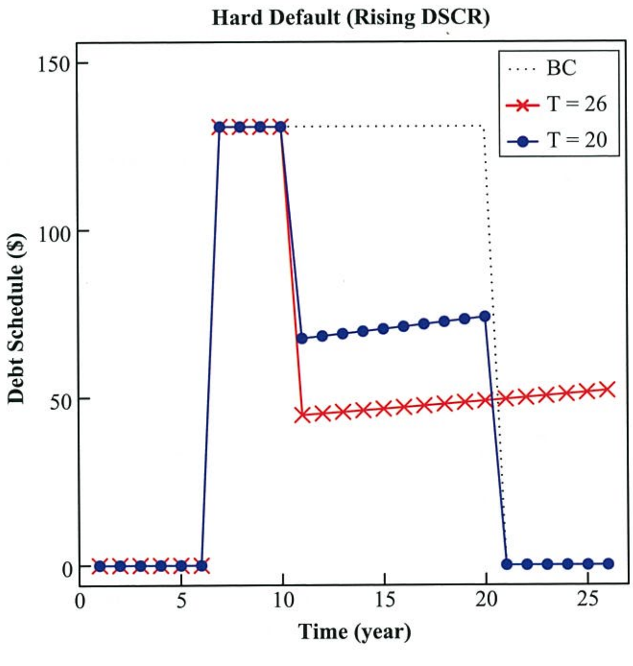


EXHIBIT 4

Debt Rescheduling upon Hard Default, BC = Base Case, T = Maturity of the New Debt Schedule



to 100%—but—creditors' risk-adjusted value may exceed that of the original loan. Upon a hard default, the post-default risk-adjusted value of the loan is given as the outcome of renegotiation process, and lenders cannot increase the value of their debt by changing the debt schedule.

Time Evolution of the Value of the Exit Option

Exhibit 5 shows the evolution of lenders' *exit value* and debt value in time. Lenders gradually shift from regions in which their exit value is relatively high (because the investment project still has many years to live) to regions in which they are less likely to choose to exit and more likely to renegotiate.

In Exhibit 5, one indicates the evolution of the total (risk-adjusted) value of debt at time *t* in both the rising (merchant) and flat (contracted) DSCR processes, and the other indicates the exit value of lenders upon a hard default (the exit value of equity holders is always 0). Finally, the third line indicates a threshold of 50% of the firm's value at time *t*, which we have assumed to be the minimum that lenders would get out of a renegotiation following a hard default.

In the case of the rising DSCR process, the creditors' exit value is higher than the 50% threshold but lower than total debt value during the first part of the firm's life; here creditors might agree to a haircut in the context of a work-out (depending on the tail size). Later, the exit value and eventually the debt value are lower than 50% of the firm value. In these cases creditors would rather take 50% of the firm value post-restructuring and are thus less likely to take a haircut (since the debt is worth less as a share of the firm's value).

Note that the likelihood that equity investors have to forgo some of their investment upon a hard default increases in time (as the lenders' decreases), since lenders take-over upon a hard default makes the outcome of renegotiation (50% for the lenders) increasingly costly for equity owners in the later stages of the investment.

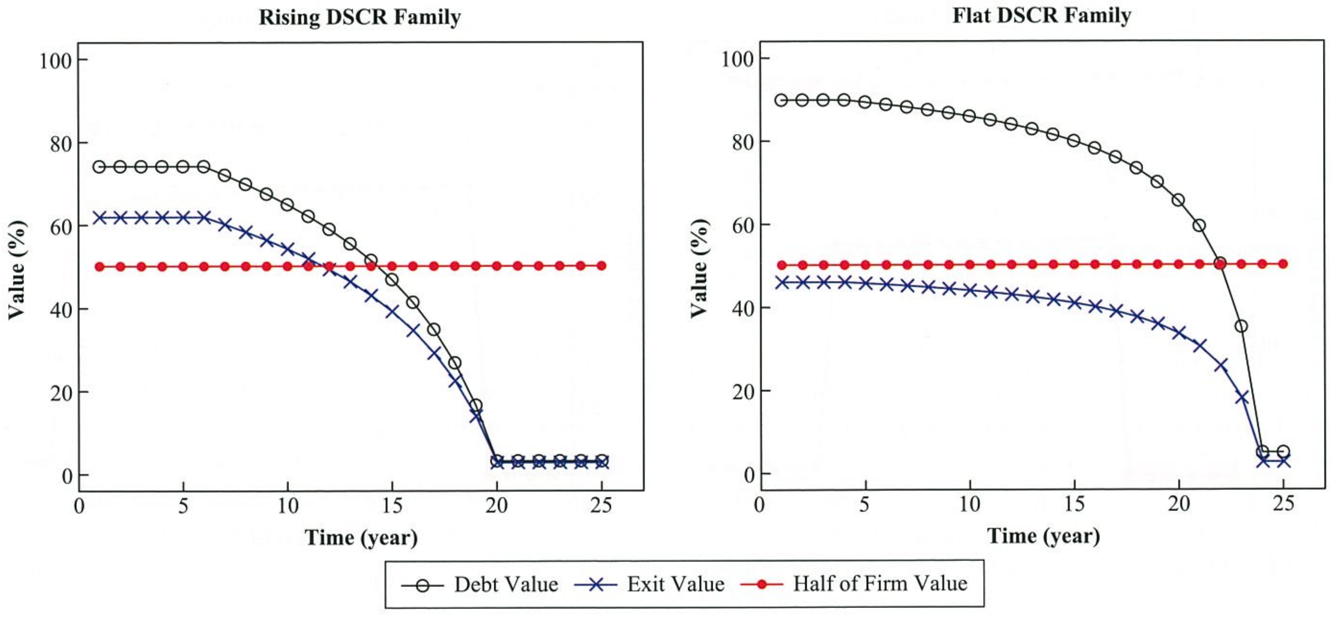
In the case of the flat DSCR process, the exit value is always less than the 50% threshold, meaning that a hard default obliges lenders to renegotiate and potentially take a haircut. However, the probability of such a haircut is very small due to the much lower probability of default implied by DSCR volatility in this case.

Risk Profile Dynamics

Next, we compare the resulting risk profiles of the two simulated DSCR processes and report expected

EXHIBIT 5

Exit Value of Lenders, and Total Firm and Debt Values



debt service, probabilities of default, per period expected losses, value-at-risk, and conditional value-at-risk (expected shortfall).

Exhibit 6 shows the expected CFADS and expected debt service, taking into account the probability of default events and their outcome, as described above. Expected debt service is lower than the base case (i.e., $t = 0$) debt service, but exceeds it in the tail, hence reducing expected losses incurred during the original loan life.

Exhibit 7 compares the probabilities of technical and hard default, as well as bankruptcy or death, i.e., the project company ceases to be a going concern upon default and there is no recovery from default. This happens when cash available upon default, including any cash in reserve accounts, exceeds the value of SPE at that stage.

The probability of (technical or hard) default decreases rapidly for the rising DSCR process since the DSCR has a rising positive mean, which makes default less likely in the later periods despite higher volatility. PD is mostly constant for the flat DSCR process since neither mean nor volatility of the DSCR process are assumed to change over time.

The difference between the probability of technical defaults and the probability of hard defaults[6] is significant in the case of flat DSCR process, making the step-in option—the right to reschedule debt upon a technical default—more valuable in the case of a flat DSCR family than a rising DSCR family.

Exhibit 8 compares the loss profile of the two DSCR processes. Expected loss (EL), VaR, and cVaR all rise continuously as maturity approaches in the case of the flat DSCR process, and VaR and cVaR tend to plateau about halfway through the original debt maturity.

The rising trend in EL can be explained by the increasing cumulative probability of default. As more hard defaults occur, creditors end up getting a haircut as post-default expected debt payments decrease. Expected debt service near the maturity of the loan reflects the accumulated effect of haircuts due to all hard defaults in previous periods. This is why mean debt payments are lower near debt maturity (as seen in Exhibit 6), and mean losses are higher, even for the rising DSCR family, for which the marginal default probability near maturity is close to zero.

The difference in the VaR and cVaR trends stems from the different tail values corresponding to the two processes. In the case of flat DSCR process, the smaller tail and relatively higher leverage near maturity of the loan increase the severity of defaults compared to earlier periods. This is because the tail is very short and

EXHIBIT 6

Expected CFADS, Base Case (T_0) and Expected Debt Service

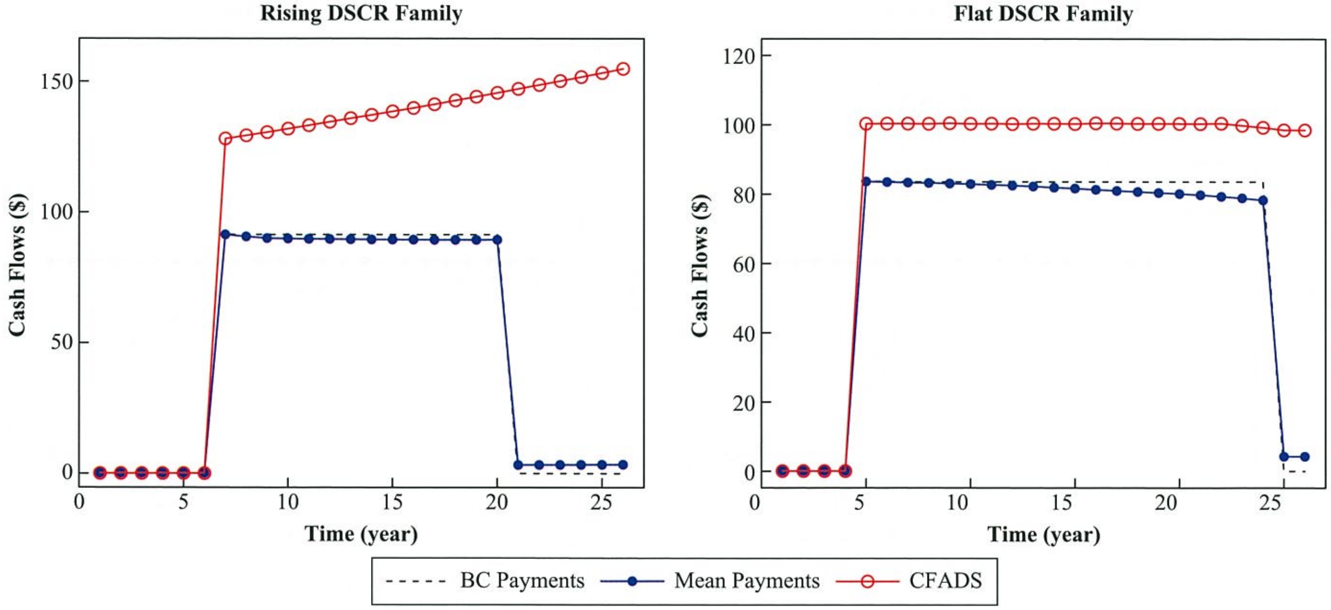


Exhibit 7

Probability of Technical Default, Hard Default (Moody's Definition) and Probability of Bankruptcy (Death)

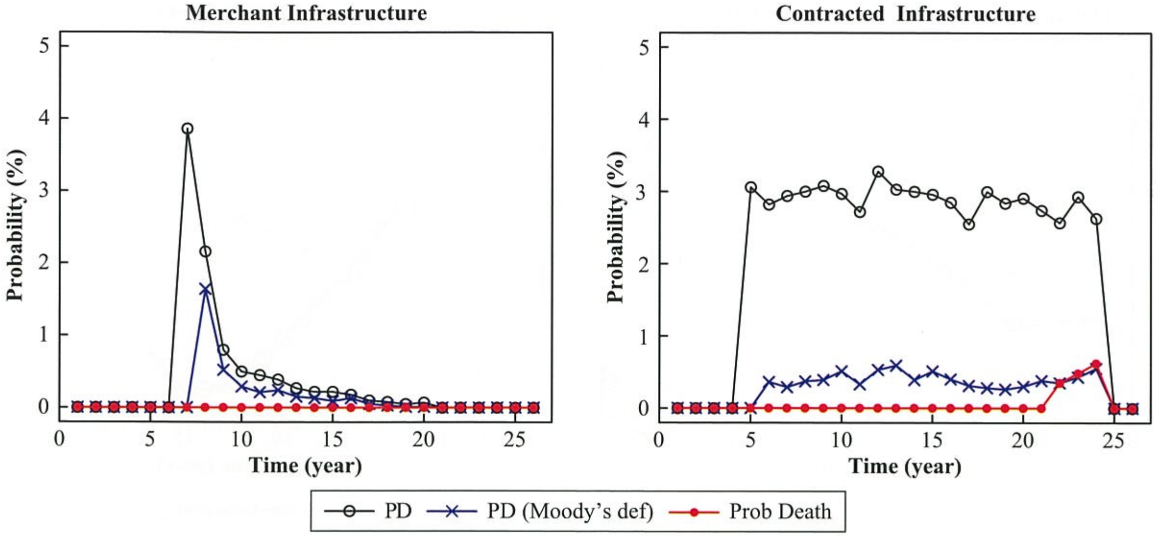
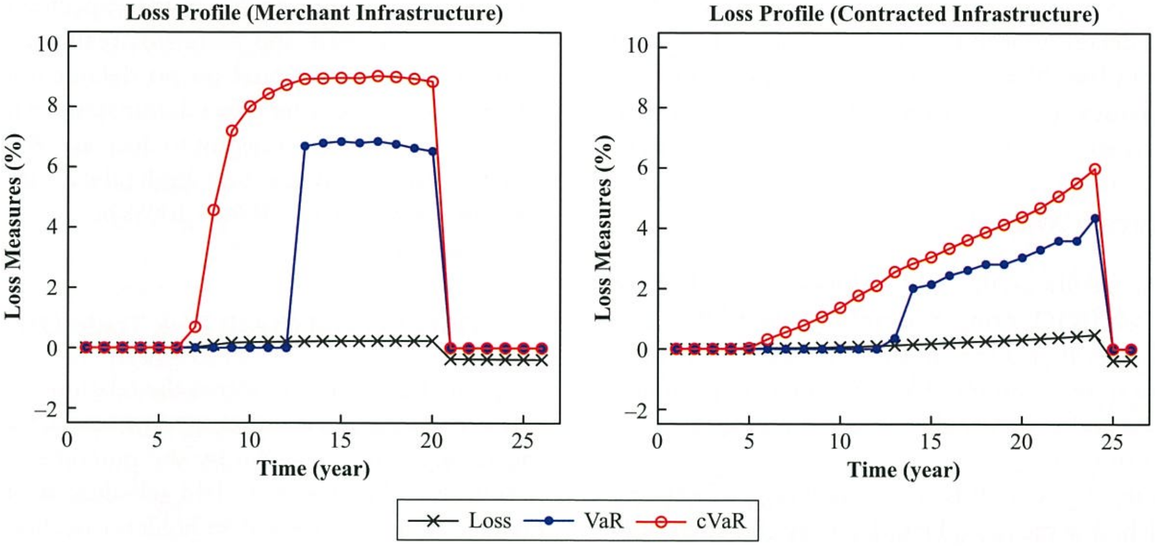


Exhibit 8

Expected Loss, VaR, and cVaR

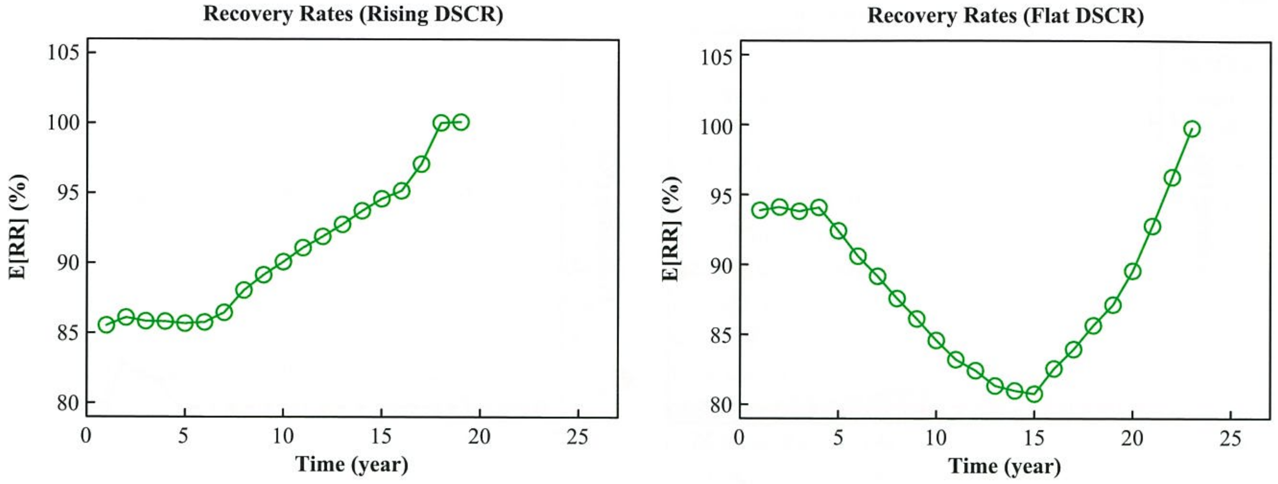


the mean CFADS stays constant. Therefore, if a hard default—however unlikely—occurs near the maturity of debt, there may not be enough cash in the tail to recover 100% of outstanding debt. That is, with this financial structure and DSCR process, defaults nearer to the maturity of the debt can be more costly than those during earlier periods.

In the case of the rising DSCR process, the tail is relatively longer and expected CFADS increases in time, creating a lot more room to recoup any potential losses. Hence, the severity of losses is much less affected by the timing of defaults.

We see the effect of different tail values further in the distribution of project deaths. Here, debt holders are better off taking the available cash and letting the firm

EXHIBIT 9
Recovery Rates



go bankrupt. In the case of the rising DSCR process, we do not see any deaths in this simulation because the extra CFADS in the tail always makes SPEs more valuable as a going concern. However, in the case of the flat DSCR process, lower tail value makes it more likely for a hard default to lead to bankruptcy near the end of the project's life. Note however that the probability of death remains very low at around 0.5%, and is non-zero only after year 22.

Recovery Analysis

Next, we discuss the time evolution of the expected recovery rate (RR) at time t, assuming that the base case debt payments are realized until $t-1$. That is, we move forward in time, assuming that base case debt payments are realized and compute the expected recovery rate at that point in time.

Exhibit 9 shows RR as a percentage of existing value of debt. For the rising DSCR process, RR generally increases in time, as the distribution of losses does not change much during the loan's life. However, in the case of the flat DSCR process, RR first decreases then increases. The decrease springs from the increasing severity of losses near the maturity of the loan, as observed in Exhibit 8, where mean EL, VaR, and cVaR all increase linearly toward the maturity of the loan. Hence, the present value of expected losses, which is affected by the full distribution of the losses and not just expected losses, increases in time as we approach the period of the most severe losses.

As we move through time, expected losses continue to increase due to the more extreme losses getting nearer, but also decrease due to the expected losses that now lie in the past and were not realized (since we compute RR_t conditional on no default until $t-1$). At some point, the latter effect dominates and the present value of expected losses begins to decrease. As expected, recovery rates are always very high (always above 85%). The final recovery rate RR_T is 100% because we assume no default until $T-1$.

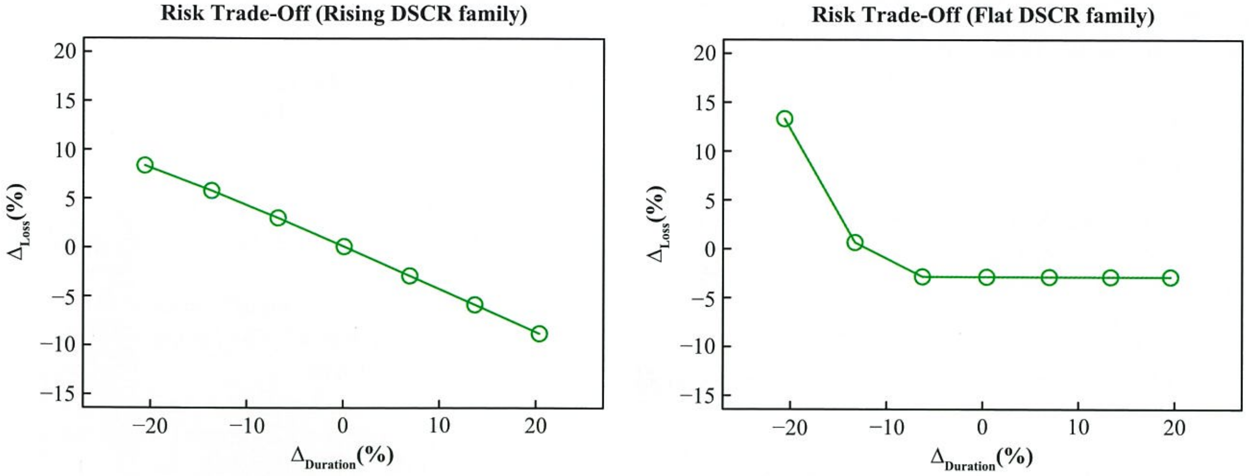
Duration and Credit Risk Trade-Off

Finally, Exhibit 10 shows the relationship between expected losses and duration upon a hard default, when the value of debt is given by the outcome of renegotiation, but the choice of debt schedule is in the debt holders' control. That is, debt holders can choose among various debt schedules that have the same risk-adjusted value at the time of default.

Both expected losses and duration are affected by the choice of new debt schedule. Each point in the exhibit is obtained by setting a maturity for the new debt schedule, and then computing the required debt service so that each debt schedule's value equals that determined by the debt renegotiation model. Those debt schedules that create lower expected losses are the ones with higher

EXHIBIT 10

Trade-Off Between Credit and Interest Rate Risk



The x-axis shows the duration relative to the mean duration, and the y-axis shows the loss relative to the mean loss.

duration, i.e., longer maturity. However, loans with very short tails embodied by the flat DSCR process do not allow trading off lower credit risk for a longer duration, only to increase credit risk for a shorter duration.

This trade-off exists because in order to reduce expected losses, the DSCR has to be kept sufficiently high, which decreases the periodic size of the renegotiated debt service and increases duration. This does not hold if exit costs were so low that debt holders would benefit from default, in which case a debt schedule with lower duration would also have a lower credit risk. In this simulation we have assumed exit costs to be high.

Hence, our model shows that there is a negative trade-off between credit risk and duration in non-recourse private project debt. With corporate debt this trade-off is typically a positive one (longer maturities imply higher credit risk).

MODEL EXTENSIONS

The framework for valuing non-recourse private project debt developed in this article and in Blanc-Brude and Hasan [2016] can further be extended to:

- Computing optimal debt repayment or DSCR profiles both *ex ante* and post-restructuring, taking into account a range of equity holders' and creditors' risk preferences.
- Designing optimal debt covenants.
- Determination of the optimal exercise of embedded options in non-recourse debt financing.

ENDNOTES

The authors would like to thank Noël Amenc, Lionel Martellini, Julien Michel, Marie Monnier, and Benjamin Sirgue for useful comments and suggestions. Financial support from NATIXIS is acknowledged. This study presents the authors' views and conclusions, which are not necessarily those of EDHEC Business School or NATIXIS.

[1] Loss-given-default (LGD) numbers reported to credit ratings agencies are also not directly comparable insofar as they are computed using lender-specific valuation methodologies.

[2] $x = 0$ corresponds to Moody's definition of default (Moody's [2015]).

[3] Equity investors may also be required to inject more capital into the project company at this stage, but we ignore this possibility and assume that outstanding debt is only paid with the free cash flows of the SPV. This is a potential extension of the model.

[4] We assume that equity owners' opportunity cost of owning the project is zero. In reality, equity holders would have to commit their time and exert effort in running the firm. Hence, their exit value (the value below which they would walk away from the project) would be the value of this time and effort spent on running a comparable alternative project. Incorporating this non-zero opportunity cost could be one of the avenues for future extensions of this model.

[5] We thank Anne-Christine Champion for raising this point.

[6] This is the Moody's definition of default: hard defaults conditional on no hard default until that time, i.e., the projects that default more than once are only counted once (Moody's [2015]).

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