An EDHEC-Risk Institute Publication

Unlisted Infrastructure Debt Valuation & Performance Measurement

Theoretical Framework and Data Collection Requirements

July 2014

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Foreword

The purpose of the present publication, "Unlisted Infrastructure Debt Valuation & Performance Measurement", which is drawn from the NATIXIS research chair at EDHEC-Risk Institute on the "Investment and Governance Characteristics of Infrastructure Debt Instruments", is to design the first academically robust, yet operationally implementable valuation and risk measurement framework for investing in illiquid infrastructure debt.

Long-term infrastructure debt poses a significant pricing challenge since market prices cannot be observed, private cash flow data is scattered amongst originators and investors, and the covenants and lender control-rights found in such instruments create *embedded options* that are not taken into account in the loan and bond valuation models typically used by lenders and investors.

Taking these characteristics into account is instrumental to capture the expected performance of infrastructure debt, in particular, its high level of post-default recovery. However, existing analyses have so far ignored the endogenous nature of credit risk in project finance and rely on *ad hoc* credit risk assumptions.

This paper is one of the stepping stones of the "roadmap" established by EDHEC-Risk Institute towards the creation of adequate performance measurement tools for long-term investors in infrastructure.

Building on advanced and robust credit risk modelling and private debt valuation techniques, this paper focuses on delivering those performance measures that are the most relevant to investors at the strategic asset allocation level, and to prudential regulators for the calibration of risk weights.

It provides a implementable framework for the formation of risk and return expectations in illiquid infrastructure debt, and also defines the most parsimonious data input requirements. Hence, we can realistically expect to deliver these performance measures at a minimal data collection cost.

In turn, the knowledge of what data needs to be collected to value infrastructure debt and derive adequate expected performance measures will help standardise data reporting for long-term investment in infrastructure.

We are grateful to NATIXIS for their support of this study in the context of the "Investment and Governance Characteristics of Infrastructure Debt Instruments" research chair at EDHEC-Risk Institute.

We wish you a thought-provoking, useful and informative read.

Frédéric Ducoulombier

Director of EDHEC Risk Institute-Asia

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Executive Summary

Executive Summary

This paper is part of an ongoing research project aiming to create long-term investment benchmarks for investors in infrastructure. It is the first valuation and risk measurement model created specifically for unlisted infrastructure debt instruments. It provides a framework to value and assess the return and risk characteristics of individual project finance loans.[1]

In a recent EDHEC-Risk Institute position paper (Blanc-Brude, 2014), we argue that improving long-term investors' access to infrastructure requires the creation of **new performance measurement tools** that can inform the asset allocation decisions of investors in infrastructure, as well as provide a sound basis for the calibration of prudential regulatory frameworks. Without the development of performance measures adapted to long-term investment in illiquid assets, investors and regulators struggle to integrate assets like infrastructure debt into their respective risk and return frameworks.

In the same paper, we describe a *roadmap* to create long-term investment benchmarks in infrastructure. We propose to address the challenges of illiquid investment performance measurement by focusing on those underlying **financial instruments that are more frequently used** in the development of new infrastructure projects, for which **tractable valuation models** can be developed that take into account their illiquid nature and can deal with the **paucity of available data**.

Indeed, measuring the performance of illiquid infrastructure investments implies two significant challenges: first, illiquidity implies that only limited information can be gleaned from market prices and, second, given the large size of each individual instrument, little private data is available to any single individual investor.

Without market prices or large cash flow datasets, performance measurement is not straightforward. But even if limited information is available for research today, it is our premise that aiming to develop the best possible knowledge of the performance of long-term investment in infrastructure — conditional on the information available today — and allowing for the possibility of learning as new data becomes available, is an improvement of the current state of *complete absence* of relevant performance measures.

In this paper, following our roadmap, we focus exclusively on private project finance (PF) loans, as they constitute by far the largest proportion of illiquid infrastructure project debt, and are well-defined since Basel-II, providing us with an uncontroversial setting to model expected cash flows.

Project finance loans are also the most relevant to long-term investors who seek to access a type of instruments previously unavailable to them (as opposed to corporate bonds), since PF is a unique form of corporate governance that creates significant and extensive control rights for lenders through *embedded options* and debt covenants.

1 - In a forthcoming paper, we develop portfolio risk and return measures, using illiquid infrastructure debt as an underlying.

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For example, debt covenants prohibiting equity holders from raising more cash through new debt or equity issuance to service existing debt can be expected to impact the default mechanism in infrastructure project finance; while debt holders' option to either restructure the debt upon default or take over the project company, can have a significant influence on expected recovery rates and the risk return profile of PF debt.

Crucially, while project finance loans are not collateralised since the investment is structured on a non-recourse or limited-recourse basis, they have a "tail" i.e. the difference between the original maturity of the project loan and the life of the infrastructure project. The free cash flow of the firm available during that period acts as a form of collateral. In certain states of the world — corresponding to a breach of covenant — lenders have control rights that allow them to restructure a loan and use its tail to maximise their recovery rate. The value of the loan's tail, as well as the relative size of liquidation and re-structuring costs of the project company, can thus be expected to have a significant impact on performance.

Because PF loans have unique characteristics, existing loan valuation models are inadequate because they not only fail to take into account the effects of debt covenants and embedded options, but also do not incorporate the dynamic nature of the credit risk profile, and often make *ad hoc* assumptions regarding probabilities of default and loss given default. Option-based valuation models used for corporate securities also cannot be directly applied to project finance loans.

If the embedded options and covenants found in PF debt are not taken into account, infrastructure debt valuation is likely to be off by an order of magnitude. In this paper, we develop an endogenous model of credit risk in order to derive more relevant and precise performance measures.

Finally, existing approaches typically fail to produce the risk-return measures that are relevant to risk management, strategic asset allocation, and prudential regulation.

Objectives and Approach

The objectives of this paper are:

1. to determine the most appropriate pricing model for infrastructure project finance loans;
2. to design a methodology that can be readily applied given the current state of empirical knowledge and at a minimum cost in terms of data collection;
3. to derive the most relevant return and risk measures for long-term debt investors: *expected loss, expected recovery rates, loss given default, value-at-risk (VaR), expected shortfall or CVaR, duration, yield, and z-spread*;
4. to define the minimum data collection requirements for infrastructure project loan valuation that can nevertheless inform a robust and academically validated pricing model.

Executive Summary

In this paper, **we show that the valuation of illiquid infrastructure project debt taking into account its illiquidity and the absence of market price feedback can be done using advanced, state-of-the-art structural credit risk modelling, relying on a parsimonious set of empirical inputs.**

In turn, following the roadmap defined in Blanc-Brude (2014), the data required to evaluate the performance of illiquid infrastructure project debt can provide **the basis for a reporting standard for long-term investors, which can also be used to populate a centralised database**, thus addressing the scattered nature of existing empirical observations, and allowing for the ongoing monitoring of the performance of long-term infrastructure investments.

As for any security, the valuation of project finance loans consists of modelling or observing cash flows and deriving their present value. However, available empirical observations are limited in time (for example a project may have a 30 year life but we cannot realistically collect more than 10 years of cash flows) and in the cross-section (each country only has so many operating toll roads or power plants relying a given contractual and financial structure).

Thus, we devise a two-step modelling process: **first, we model the cash flows of generic types of financing structures** that are commonly found in infrastructure project financing.

Thus, by partitioning the investable universe of infrastructure projects into tractable cash flow models characterised by well-documented parameters – such as initial leverage, amortisation profile, and typical average debt service cover ratio throughout the project lifecycle – we can identify reasonably **homogenous families of project structures**, which we can be considered to correspond to a *single* underlying cash flow process.

Second, given a generic cash flow model, we build a valuation model to derive the relevant return and risk measures. This model takes into account the fact that illiquid markets with large transaction costs – as is the case of infrastructure project debt – do not lead to the formation of unique prices, or valuation measures, but instead that the value of the same asset must lie within a range determined by the characteristics and preferences of individual investors.

Thus, our methodology also determines "arbitrage bounds" or limits on possible valuations for illiquid infrastructure debt which asset values can be expected to lie.

Next, we describe each step in more details, before presenting our main results.

Cash Flow Model

The task of projecting future cash flows to project finance lenders first requires to determine the future free cash flows to the project company before deriving cash flows to lenders.

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The free cash flows of the project company — often referred to as the Cash Flow Available for Debt Service or CFADS — is private information and not easily observed. Instead, we focus on the Debt Service Coverage Ratio (DSCR), which is typically monitored and recorded by infrastructure PF lenders. Indeed, knowledge of the distribution of the DSCR at each point in time, combined with the Base Case Debt Service, which is also easily observable at the time of financial close, can be used to infer the expected value and volatility of the CFADS of a typical infrastructure project.

In this paper, we focus on two generic project types, which embody a large number of real-world infrastructure projects and their associated debt securities. We define two families of DSCR dynamics called 1) merchant infrastructure and 2) contracted infrastructure.

Merchant infrastructure refers to those projects that generate revenue by selling their output or service in a market, and hence are exposed to market risks, while **contracted infrastructure** projects receive a contracted revenue stream in exchange for providing a pre-agreed output or service, and bear little to no market risk.

Examples of merchant infrastructure projects may include a power plant that sells electricity at market prices or a road collecting tolls from users. Examples of contracted projects may include schools and hospitals that receive a fixed payment from a government entity upon the satisfactory delivery and maintenance of an infrastructure, or an energy project financed on the back of take-or-pay purchase agreement.

These two project types have different underlying business risk, and *as a consequence*, they are financially structured in different manners. Merchant infrastructure projects are generally structured with a rising mean DSCR, and a longer tail. A rising DSCR implies that lenders get paid faster than the equity owners, and a longer tail increases the value of lenders' security. In other words, lenders demand an increasing mean DSCR and a longer tail to protect themselves against a higher and increasing DSCR volatility, which results from higher revenue risk.

In contrast, contracted projects are structured with a flat DSCR and shorter tails, as lenders demand less protection against default due to lower expected underlying revenue risk.

Of course, other generic project financial structures exist, even though they tend to be a combination of these two types, e.g. *shadow* toll roads collect a volume-based income paid typically by a government.

For each generic type, we initially model the CFADS of a generic project financing structure, using typical values for initial leverage, tail length, contracting periods, etc. and reasonable parameter estimates of the DSCR. In due course, once enough empirical observations become available, DSCR parameters can be updated using Bayesian inference techniques as suggested

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in Blanc-Brude (2014) and detailed in Blanc-Brude and Hasan (2014).

Once the future CFADS distribution is known, projecting cash flows to debt holders is possible if the debt schedule is also known. But while one debt "base case" schedule is determined at financial close and is known *ex ante*, we know that restructurings or "work outs" following a breach of covenant are common in project finance, which can change the debt schedule. Thus, we need to model these changes in the debt schedule to be able to determine total cash flows to lenders in all possible states of the world.

To take into account these potential changes in the debt schedule, we model the debt renegotiation process to determine the outcome of restructuring after either a technical (covenant-driven) or a hard default (of payment). With technical defaults, lenders can only try to maximise the recovery rate of the original debt given the tail, whereas hard defaults give them more options, including exiting the relationship with original equity investors and taking over or selling the project or its debt.

The new debt service is determined by taking into account what each party would lose in the absence of a workout. In our dynamic renegotiation model, if there is no space for renegotiation upon a hard default, the project is taken over by lenders (which may seek a new equity investor). Conversely, restructuring must take place as long as the value of either debt or equity post-restructuring is higher than in the absence of renegotiation i.e. as long as lenders can get more than their *exit value* and equity holders more than nothing, a new debt schedule is agreed and maximises recovery in the tail.

Valuation Model

Thus, given a model of expected cash flows taking into account the conditional distribution of the DSCR at time t and the outcome of renegotiations between debt and equity holders upon technical and hard defaults, we can determine the cash flows to project finance lenders in every future state of the world.

To value these cash flows, we take a so-called structural approach. Structural models present the advantage of calculating the value of firm's securities as a function of their fundamentals. Credit risk measures, such as the probability of default, loss given default, value-at-risk &c are determined by an explicit mechanism corresponding to a value threshold, instead of being exogenously specified.

In project finance, the thresholds that lead to credit events are well defined as a function of the DSCR and monitored, that is, observable, which is a substantial improvement on most structural credit models applied to regular corporate debt. In particular, we show that **distance to default** can be expressed as a function of the distribution of the DSCR.

Most cash flow discounting models use a risk premium to be added to the time

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value of money (the risk-free rate) in order to compute a value. However, in structural models of credit risk, the heterogeneity of investors' preferences is incorporated through risk-adjusted or risk-neutral probabilities. For risk-averse investors, risk-neutral probabilities penalise future cash flows by decreasing their expected value under the equivalent risk-neutral measure.

That is, instead of discounting actual expected cash flows at a premium to the risk-free rate, they are decreased under the risk neutral measure and then discounted at the risk-free rate. The more risk-averse an investor, the higher the premium demanded for each unit of risk, and the lower the expected value under the risk-neutral probability measure.

This technique is routinely used in option pricing models: the required price of risk (and the risk-neutral probabilities) are determined such that the expected present value of the risky asset's cash flows under the risk-neutral measure is equal to the observed market price of a portfolio with an equivalent payoff.

In the absence of market prices however, as is the case with illiquid infrastructure debt, there is no unique value to which the discounted risk-adjusted cash flows should correspond. Instead, incorporating investors' risk preferences to determine the value of expected cash flows leads to a range of values, since the required price of each unit of risk must depend on individual investors' unique circumstances, including regulatory requirements, the diversification level of the existing portfolio or the structure of their liability.

In this case, we argue that the required price of one unit of risk (the required Sharpe or reward-to-risk ratio) should always lie in an *approximate arbitrage band* of $[0, 2]$ that rules out investments that are either too risky for any investor to take, or too attractive not to be arbitraged away despite the illiquidity of these instruments.

The lower limit of the band corresponds to an investor that requires no premium above the risk-free rate for bearing the risks in PF loans. This could be the case for risk-neutral investor. The upper limit corresponds to an investor that requires a premium of 200 basis points for bearing each unit of risk (one standard deviation of the DSCR) taken in a PF loan.

The combination of both cash flow and valuation models allows us to evaluate the performance of project finance loans from the perspective of different individual investors.

Finally, the value of expected cash flows under the risk-neutral measure can be determined using the Black-Cox decomposition, which divides a security's cash flows into four components: 1) its payout at maturity, 2) its payout if the debt reorganises at a lower boundary i.e. default, 3) its payout if the debt is restructured at an upper boundary i.e. refinancing, 4) its payout before reaching any of these boundaries.

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The present value of these four payouts determines the total value of a project finance loan at each point in its life given all the paths that the debt cash flows can take in different states of the world.

Results

Here, we report results for a typical investor requiring a Sharpe ratio of 1 to invest in illiquid infrastructure debt and typical parameters detailed in chapter 4.

A low but dynamic risk profile

We find that the debt of both types of generic infrastructure projects discussed in this paper — merchant and contracted — exhibit highly dynamic risk profiles.

In the case of merchant infrastructure projects, the probability of both technical and hard defaults (PD), and of hard defaults only (Moody's definition) shown on figure 1, goes down sharply post construction, while expected loss (EL) and extreme losses (VaR,cVaR) tend to rise throughout the loan's life. Similarly, in the case of contracted infrastructure projects, while PD stays almost constant during the loan's life, the severity of losses increase with time.

The diverging trends in the distribution of defaults and losses are a consequence of restructurings upon defaults. Even if defaults are concentrated in a certain period of time, debt restructuring can spread losses over the entire life of the project. Hence, losses tend to increase with time, as the cumulative number of defaults (and hence restructurings) accrue losses near the end of loan's life. However, part of the losses suffered during the loan's life are , recovered in the loan's tail, thus reducing the overall expected loss.

Indeed, risk levels are found to be relatively low and recovery relatively high. While EL never rises above 2%, VaR and CVaR while they increase towards the end of the loan's life as the value of the tail is exhausted, never reach levels higher the 6% and 10% respectively, while expected recovery rates are always in the 80% to 100% range, as shown on figure 2.

Hard default frequencies match reported averages

The different aspects of the projects' risk profile can largely be explained by their DSCR profiles, tail values, and the costs of exit relative to the cost of renegotiation for lenders.

The rising DSCR profile of merchant infrastructure implies that the project's likelihood of default decreases faster in time. If a loan survives the first few years after the construction stage, the increasing mean DSCR more than offsets the increasing DSCR volatility, making it less likely that the project will default in the future. For contracted infrastructure, flat DSCR profile implies that the probability to default barely changes in time, though it stays at a very low level due to lower DSCR volatility.

Moreover, when using Moody's definition of default in project finance — by which each loan is only allowed to default once

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Figure 1: Comparison of probability of technical or hard default (PD), hard default only (Moody's definition) and probability of death (no recovery), for the two DSCR families.

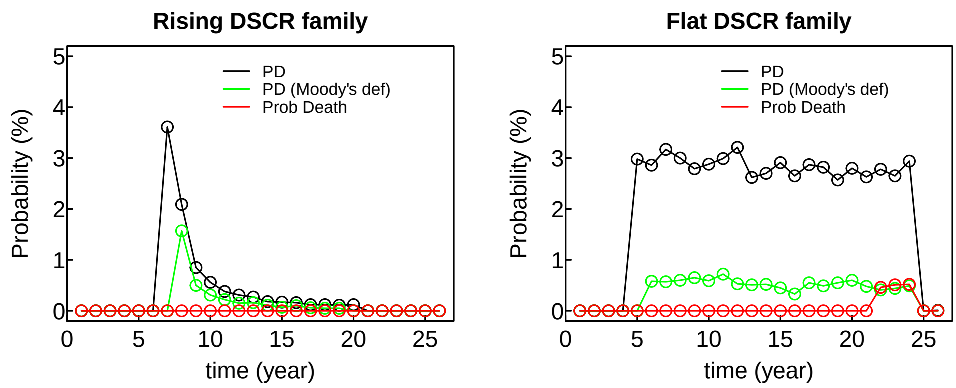

Figure 2: Comparison of expected loss, VaR, and cVaR for the two DSCR families.

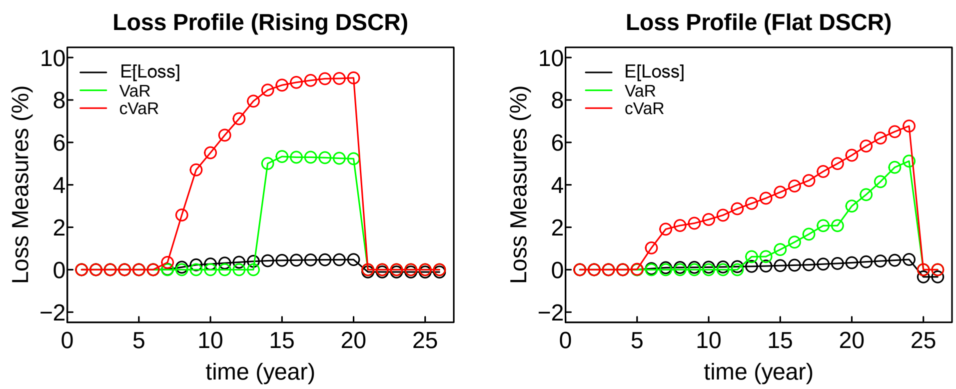

Figure 3: Loss given default (present value of expected losses) as a percentage of the value of debt.

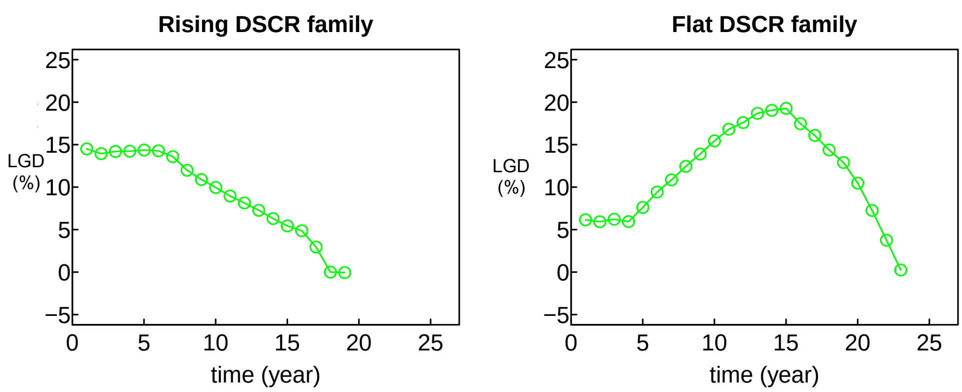

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Figure 4: Traded-off between credit and interest rate risk. The x-axis shows the duration relative to the mean duration, and the y-axis shows the loss relative to the mean loss.

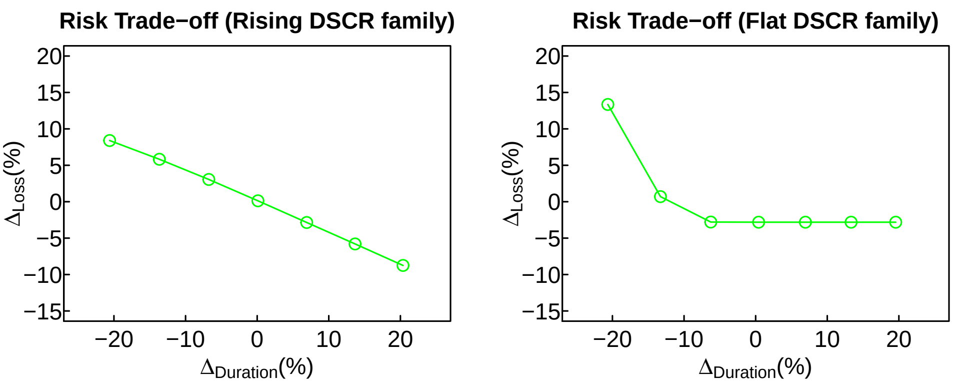
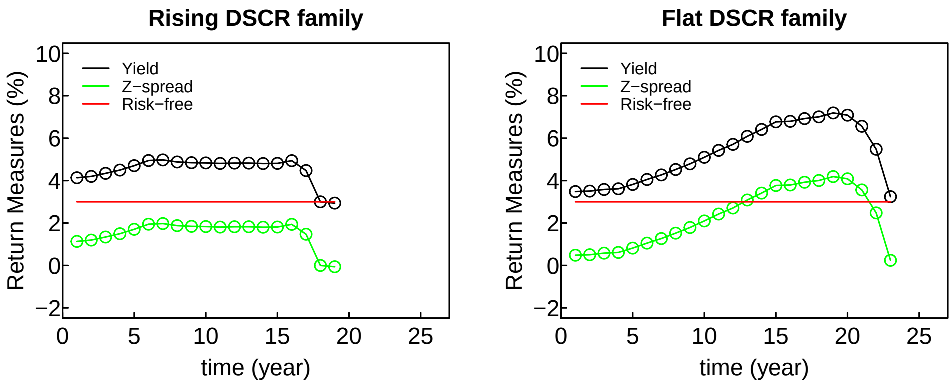


Figure 5: Comparison of yield, and z-spread for the two DSCR families.



(Moody's, 2013) – we find marginal default frequencies in line with reported empirical estimates, trending downwards from just under 2% at the beginning of the loan's life to almost zero after ten years, in the case of merchant infrastructure, and flat at 0.5% for contracted projects.

While Moody's (2013) does not explicitly differentiate between merchant and contracted projects, its main sample is effectively dominated by merchant or part merchant projects, yielding the oft-reported decreasing PD profile reproduced here on page 32; while in a separate study focusing on PPPs – effectively contracted infrastructure – Moody's report very low PD in the range predicted by our model.

Low credit risk and high recovery

The loss profiles for the two DSCR families shown on figure 2 are similar insofar as expected losses (EL) are very low and then increase towards the maturity of the loan, but differ in terms of the behaviour of extreme losses. Extreme losses (VaR and cVaR) increase almost linearly towards the maturity of the loan for contracted

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infrastructure projects, but stay relatively constant near the loan's maturity for merchant projects.

The increasing EL for both DSCR families is a consequence of cumulative haircuts received upon hard defaults in all the prior periods. The increasing VaR and cVaR in the case of flat DSCR family are due to a lower tail value, and constant leverage in time, the combination of which implies that near the loan's maturity the remaining value of the project may not be sufficient to offset losses, making defaults more severe.

Figure 3 shows the evolution of the loss given default (LGD) i.e. one minus the recovery rate, as a function of time. Recovery rates are very high (always above 85%).

For merchant infrastructure (rising DSCR), LGD decreases in time, as the distribution of losses does not change much during the loan's life. For contracted infrastructure however (flat DSCR), the LGD first increase, and then decrease.

This increase in LGD for the flat DSCR family arises from the increasing severity of losses near the maturity of the loan as observed in figure 2: mean EL, VaR, and cVaR all increase linearly towards to maturity of the loan. Hence, LGD, which is affected by the full distribution of the losses and not just by mean losses, increases in time as we approach the period of the most severe losses. As we move through time, expected losses continue to increase due to the more extreme losses getting nearer, but also decrease due to the potential losses that now lie in the past and were not realised. At some point, the latter effect dominates and LGD begin to decrease.

Value is driven by lenders' exit option and monitoring

Importantly, the size of losses for both DSCR families is primarily influenced by lenders' exit value net of exit costs. Exit costs determine the aggregate loss of value (debt+equity) if the debt owners take over the project company upon a hard default and do not renegotiate with the original equity investors.

The higher the exit costs, the lower the value that lenders can obtain by taking over the project company after a hard default, and the lower their bargaining power in negotiations with original equity holders.

This is primarily a consequence of the unsecured nature of project finance debt, which makes the value of project company strongly dependent on the owners' ability to run it. In the absence of expertise required to run the project company, the lenders are likely to be forced to offer concessions to equity holders to benefit from their ability to run the firm. Hence, lenders may have to suffer losses even in otherwise low risk projects like contracted infrastructure because replacing the equity owners upon a hard default, while it is in their power, can be very costly in some cases.

As a consequence, ongoing monitoring of the SPE conducted is required of lenders in project in order to avoid ever having to

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contemplate exercising their option to exit, in particular, technical default triggers (e.g. a low DSCR or loan-life cover ratio) allow lenders to intervene and maximise their recovery rates long before more expensive options to restructure, sell or liquidate the SPE ever arise.

A DSCR-driven yield profile

The yield curve for both types of project debt is driven by two forces: the increasing severity of losses towards the end of the loan's life pushes up the yield since the discounted value of expected cash flows is further reduced, while the sequential resolution of uncertainty as maturity approaches pulls it down. The actual yield curve shown on figure 5 balances the two effects.

Initially the yield goes up as we get closer to the region where larger losses are likely to be accrued and the first effect dominates. However, as we move past this region, the probability of default during the remaining life of the loan goes down and expected recovery goes up: at one point the yield starts to decrease, as the second effect begins to dominate. In the case of rising DSCR projects, for which PD decreases more sharply and losses are more evenly distributed, uncertainty is resolved faster, and the yield begins to go down sooner in the project lifecycle.

A credit vs. duration risk trade-off

Finally, we also illustrate how the ability to reschedule debt upon technical and hard default creates a trades off between credit risk and duration risk. That is, to reduce the credit losses upon default, investors have to extend the maturity of their loan further in the tail, and have to bear a higher interest rate risk due to a higher duration. This trade-off can be quantified, as shown on figure 4, and may help determine the optimal debt schedule for an investor with a given aversion to credit and interest rate risks.

Next Steps: Data Collection and Portfolio Construction

Thus, with a parsimonious set of inputs that consists of the parameters of the DSCR distribution across different types of generic projects, the base case debt schedule and a number of variables defined in the covenants at financial close, infrastructure project finance loans can be valued at any point in time, and their risk/return profile can be constructed spanning the entire life of the loan.

In other words, by partitioning the infrastructure project finance universe into a parsimonious set of tractable cash flow models, which can be calibrated using available data in due course, we can create the *building blocks* thanks to which the systematic performance of different exposures to infrastructure debt can be identified, and later portfolio (benchmark) construction can take place.

In this paper, we deliver the first three steps of the roadmap defined in Blanc-Brude (2014) with respect to infrastructure debt investment: defining the most relevant underlying financial instrument, designing a

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valuation framework that is adapted to its private and illiquid nature, and the determination of a standard for data collection and investment performance reporting in infrastructure investment.

Next steps include active data collection to better calibrate our model of $DSCR_t$ dynamics, before moving to the portfolio level of the analysis, towards long-term investment benchmark in infrastructure debt.

1. Introduction

1. Introduction

Both long-term investors and prudential regulators have become increasingly aware of growing investment opportunities in illiquid infrastructure debt in recent years. However, a valuation framework allowing the derivation of adequate return and risk measures for this type of instruments has remained elusive. Indeed, a specific valuation framework is needed, first because infrastructure project debt has unique characteristics not found in standard corporate credit instruments, and second because illiquidity implies a high degree of data paucity, which needs to be explicitly addressed in the valuation approach.

In this paper, we develop the first valuation and risk measurement framework for unlisted infrastructure project debt instruments.

We focus exclusively on private project finance loans, as they constitute by far the largest proportion of illiquid infrastructure project debt. They are also the most relevant to long-term investors who seek to access a type of instruments previously unavailable to them (as opposed to corporate bonds).

Indeed, project finance (PF) is a unique form of corporate governance, which creates significant and extensive control rights for lenders through embedded options and debt covenants.

For example, debt covenants prohibiting equity holders from raising more cash through new debt or equity issuance to service existing debt can be expected to impact the default mechanism. Likewise, debt holders' right to either restructure infrastructure project debt upon default or liquidate the project company, can have a significant influence on expected recovery rates and the risk/ return profile of PF debt.

If such options and covenants are not taken into account, infrastructure debt valuation is likely to be off by an order of magnitude. In this paper, we develop an endogenous model of credit risk in order to derive more relevant and precise performance measures.

Our paper also aims to address the more prosaic difficulties introduced by the absence, not only of market prices for unlisted infrastructure project debt, but also of any large database of infrastructure debt cash flows.[2] Infrastructure projects are long-lived and today complete time series of cash flows spanning several decades are not available to researchers or investors.

Standalone investable infrastructure projects are also lumpy and only number in the thousands globally over the past 15 years: observations are also limited in the cross-section.

Crucially, the endogenous nature of credit risk in project finance implies that a limited number of events of default and recovery can be observed, making frequency-based approaches to credit risk measurement inconclusive. Instead, as we argue in Blanc-Brude (2014), Bayesian inference can be used to calibrate cash flow models of generic project financings and derive

2 - While rating agencies collect information on individual events of default, they only aim to measure credit risk – the likelihood of default – not to derive investment return or risk measures.

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performance measures to the best of our current knowledge.

But before calibrating posterior cash flow distributions, we must first build a valuation framework that can meet the objectives described in the next section.

1.1 Objectives of This Paper

The objectives of this paper are:

1. to determine the most appropriate pricing model for infrastructure project finance loans;
2. to design a methodology that can be readily applied given the current state of empirical knowledge and at a minimum cost in terms of data collection;
3. to derive the most relevant return and risk measures for long-term debt investors and regulators: *expected loss, expected recovery rates, loss given default, value-at-risk (VaR), expected shortfall (cVaR), duration, yield, and z-spread*;
4. to define the minimum data collection requirements for the purpose of infrastructure project loan valuation, that can nevertheless inform a robust and academically validated pricing model.

Next, we provide a more detailed justification of our choice of definition of underlying infrastructure debt.

1.2 Defining Infrastructure Debt

What constitutes "infrastructure" is and will likely remain a matter of debate. For our purpose, any definition of infrastructure debt is a matter of trade-off between clarity and comprehensiveness.

Our proposed choice is first determined by the requirement to have a clear definition of underlying instruments in a context where, because of data paucity, we must rely on *ex ante* cash flow models that can later be calibrated to the best of our current knowledge with existing and available empirical observations.

Because infrastructure project finance is well-defined since Basel-II,[3] it provides us with an uncontroversial setting to model expected cash flows, using input parameters for generic project financing structures which are transparent and can be the object of an industry consensus.

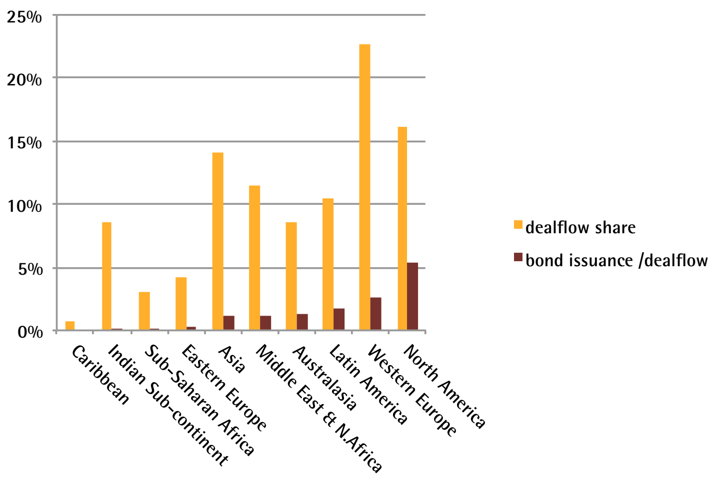
Our focus on project finance is also warranted because most infrastructure investment and the immense majority of new or `greenfield' infrastructure projects are delivered via project financing. We estimate that more than USD3.3Tr of project financing was closed worldwide between 1995 and 2013.

Private loans constitute the lions' share of total infrastructure project debt (Yescombe, 2002). As figure 6 illustrates, bond financing has always played a minimal role in project finance globally. In North America, where project bonds are the most used, cumulative issuance between 1994 and 2013 amounts to a mere 5% of the total deal flow. The figure is much lower in other regions. Thus, it is fair to say that **the immense majority of**

3 - "Project finance is a method of funding in which investors look primarily to the revenues generated by a single project, both as the source of repayment and as security for the exposure. In such transactions, investors are usually paid solely or almost exclusively out of the money generated by the contracts for the facility's output, such as the electricity sold by a power plant. The borrower is usually a Special Purpose Entity that is not permitted to perform any function other than developing, owning, and operating the installation. The consequence is that repayment depends primarily on the project's cash flow and on the collateral value of the project's assets." (BIS, 2005)

1. Introduction

Figure 6: Regional shares of the cumulative project finance deal flow and project bond issuance, 1994-2013



Source: Dealogic, authors' calculations

infrastructure project financing consists of private loans.

Hence, by focusing on project finance debt i.e. unlisted senior loans extended to special purpose entities (SPEs) on a limited- or non-recourse basis, we capture the bulk of private infrastructure financing and gain a clear definition of infrastructure debt as an underlying instrument. This is instrumental since our purpose is to discuss infrastructure investment on a scale that is congruent with institutional investing i.e. implying substantial asset holdings. To achieve a degree of generality in our conclusions, we choose to focus on the most representative type of instrument used in infrastructure debt finance.

1.3 Proposed Approach

As for any security, the valuation of project finance loans consists of modelling or observing cash flows and deriving their present value. However, empirical observations are limited in time (for example a project may have a 30 year life but we cannot realistically collect more than 10 years of cash flows) and in the cross-section (each country only has so many operating toll roads or power plants). Blanc-Brude (2014) provides a detailed discussion of data limitations for the purpose of long-term infrastructure investment benchmarking.

Thus, we devise a two-step process: first, we model the cash flows of generic types of financing structures that are commonly found in infrastructure project financing; next, given a generic cash flow model, we build a valuation model to derive the return and risk measures listed in section 1.1.

1.3.1 Cash Flow Model

The task of projecting future cash flows to project finance lenders requires first to estimate future free cash flows to the SPE, and second to determine the cash flows to lenders.

1. Introduction

The free cash flows of the SPE — often referred to as the Cash Flow Available for Debt Service or CFADS — is not easily observed, mostly due to the private nature of the project company. Instead, we focus on the Debt Service Coverage Ratio (DSCR), which is typically monitored and recorded by lenders. Indeed, knowledge of the distribution of the DSCR at each point in time, combined with the Base Case Debt Service (DS^{BC}), can be used to infer the expected value and volatility of the CFADS of a typical SPE.

Once the future CFADS distribution is known, projecting cash flows to debt holders is possible if the debt schedule is also known. But while one debt schedule is determined at financial close, and is known *ex ante*, restructurings[4] are common in project finance which can change the debt schedule. Thus, we need to model these changes in the debt schedule to be able to determine the total cash flows to lenders.

4 - In this paper, we use the terms debt restructuring and debt reorganisation interchangeably

To model these changes in the debt schedule, we use a game theoretic model to determine the outcome of restructurings during financial distress: the new debt schedule is determined primarily by the total value of the project company at the time of reorganisation, and the relative bargaining power of debt and equity holders, which is determined in part by debt covenants, and in part by debt holders' option to take over upon default.

In this paper, we initially model the CFADS of generic project financing structures for reasonable parameter estimates. In due course, once enough empirical observations become available, these parameters can be updated using Bayesian inference techniques as suggested in Blanc-Brude (2014).

1.3.2 Valuation Model

Thus, given a model of expected cash flows taking into account the conditional distribution of the DSCR at time t and the outcome of renegotiations between debt and equity holders, we can determine the cash flows to project finance lenders in every state of the world.

The valuation model requires incorporating investors' preferences towards risk to determine the cumulative value of expected cash flows. Most cash flow discounting models use a risk premium to be added to the time value of money (the risk-free rate) in order to compute a value.

Instead, we integrate investors' risk aversion in the cash flow distribution so that we can always discount them at the risk-free rate i.e. instead of adding the cost of risk to the denominator in the discounting formula, we take into account its effect in the numerator. The resulting risk adjusted cash flow distribution is referred to as the risk-neutral distribution, and the determination of the risk neutral probability measure primarily requires the determination of the required premium for one unit of risk.

This technique is routinely used in option pricing models: the required price of risk (and hence the risk neutral probabilities) are determined such that the expected present

1. Introduction

value of the risky asset's cash flows under the risk neutral measure is equal to the observed market price.

In the absence of market prices however — as is the case with illiquid infrastructure debt — there is no unique value to which the discounted risk-adjusted cash flows should correspond. Instead, the required price of each unit of risk can depend on individual investors' unique circumstances, including regulatory requirements, the diversification level of the existing portfolio and the structure of their liabilities.

We argue that the required prices of risk always lie in an 'approximate arbitrage band' of $[0, 2]$ that rules out investments that are either too risky for any any investor to take, or too attractive to survive in the market. The lower limit of the band corresponds to an investor that requires no premium above the risk free rate for bearing the risks in PF loans. This could be the case for a very well diversified investor, for whom the marginal contribution of the loan to the portfolio risk may be zero e.g. the State. The upper limit corresponds to an investor that requires a premium of 200 basis points for bearing each unit of risk (one standard deviation of the DSCR) taken in a PF loan.

We argue, from both a theoretical and an applied perspective, that Sharpe ratios above this upper limit would to be too good to be true ("good deals") and thus cannot exist. [5]

5 - Alternatively it could be argued that they may only exist in cases where the Sharpe ratio does not capture the full risk of the investment.

Finally, we posit that underlying cash flow process can be decomposed into two components: 1) a component that is correlated with the traded securities, and 2) a component that is uncorrelated with the traded securities. This is identical to treating PF loan as a combination of a traded and an untraded portfolio, with the bounds discussed above only applying to the untraded part.

The combination of both cash flow and valuation models allows us to evaluate the performance of project finance loans from the perspective of different individual investors.

1.4 Structure of This Paper

The rest of this paper is structured as follows: chapter 2 discusses the characteristics of infrastructure project finance debt and the need to design a model of endogenous credit risk and recovery. Chapter 3 discusses existing valuation methodologies and the relevant literature. It then provides an intuitive overview of our proposed valuation framework. Chapter 4 details the implementation of the model for two generic types of infrastructure projects. In chapter 5, we present the resulting risk and return measures of illiquid infrastructure project debt. Chapter 6 summarises and discusses our findings.

2. Characteristics of Private Infrastructure Debt

2. Characteristics of Private Infrastructure Debt

In this chapter, we review the characteristics of infrastructure project finance debt and further discuss why an endogenous model of credit risk and recovery is necessary to value such instruments.

2.1 Observable Asset Value

Project financing amounts to investing in a single-project firm or Special Purpose Entity (SPE) with a pre-defined lifespan. Before the financing decision can be taken, this SPE has to demonstrate its financial viability with a high degree of probability.

In project financing, as opposed to traditional corporate finance, the free cash flow of the firm is the main determinant of asset value. At any time t during the SPE's finite life, the firm's value is simply the sum of expected Cash Flow Available for Debt Service or CFADS, discounted at the appropriate rate. This value is the only quantity against which the SPE may initially borrow (and later re-structure or re-finance) any debt.

In the majority of cases, the project SPE does not own any tangible assets,[6] or owns assets that are so *relationship-specific* that they have little or no value outside of the contractual framework that determine the future CFADS stream, and justifies the investment in the first place.

Project financing also means that the owners of the SPE provide very little, if any collateral to secure its debt. In project finance, contracts must suffice to create enforceable and valuable claims and to define expected cash flows with reasonable accuracy (see Blanc-Brude, 2013, for a discussion).

The only form of collateral available to lenders is known as the loan's "tail" i.e. the SPE's cash flow available for debt service beyond the original maturity of the loan, and over which lenders have control rights in states of the world embodied by certain covenant breaches.

Hence, unlike traditional firms, the value of the total assets of an SPE can be observed. This makes structural credit risk models, which derive a firm's credit risk from its total asset value, a natural and more suitable choice than reduced form models, which consider default a random event. We return to this point in more detail in section 3.1.2.

The CFADS thus plays a central role in our approach to value infrastructure debt: it is the risky (stochastic) *underlying process* driving value in project finance debt securities, not dissimilar to the stochastic processes referred to in the design of option pricing formulas.

In this context, an important feature of project finance is the role of *initial* financial leverage (agreed at financial close). In a recent review, we report that senior leverage[7] in infrastructure project finance consistently averages 75% between 1994 and 2012, irrespective of the business or credit cycle, and can be as high as 90% for certain categories of projects with a the most predictable free cash flow (Blanc-Brude and Ismail, 2013).

6 - In the most frequent case of public infrastructure projects financed through a so-called public-private partnership contract, the ownership of the tangible infrastructure assets remains *de facto* and, most often, *de jure* in the public domain.

7 - The ratio of senior debt to total investment.

2. Characteristics of Private Infrastructure Debt

Indeed, we and others have argued that the high leverage typically observed in project finance should be interpreted as a sign of *low* asset risk (Esty, 2003; Blanc-Brude, 2013) i.e. lenders agree to provide most of the funds necessary to carry out the planned investment without further recourse or security because the probability of timely repayment is considered to be very high.

Beyond the predictability of the SPE's business model and therefore its ability to meet the base case debt service agreed at financial close, lenders agree to extend the majority of the necessary funds because of the covenants and embedded options that are found in project finance debt and that create unique state-dependent control rights for them, of the sort that are not found in traditional corporate debt.

2.2 Covenants and Embedded Options

Because project finance SPEs typically have a high degree of initial leverage, debt contracts often contain covenants to protect debt holders. These covenants can vary from one loan to another, depending on the bank's relationship with the counter-party and the bank's assessment of the project's risk.

Nevertheless, covenants and embedded options commonly found in project finance debt include (see Yescombe, 2002):

- **Minimum Debt Service Coverage Ratio (DSCR) requirement**: In order to mitigate credit risk, debt covenants often require the borrower to maintain at least a minimum level of the debt service coverage ratio – the ratio of the free cash flow of the SPE to the current period's scheduled debt payment. If the DSCR falls below a pre-agreed threshold, equity dividends can be "locked-up" to create a supplementary cash buffer for debt holders, as well as to create incentives for equity investors to resolve the problems that have led to lower than expected free cash flow (to the extent that it is in their power).

- **Non-financial default triggers**: In addition to covenants that trigger default due to financial weakness (missed debt payment, or a decrease in the DSCR below the minimum stipulated level), default can also be triggered by non-financial or operational events. For example, events such as the revocation of the SPE's license to perform a business, or the failure to complete construction in time, or the default of a counter-party to the SPE, can lead to an event of default. Once this has occurred, the project cannot be managed without lenders' involvement (see Yescombe, 2002, section 13.11).

- **Step-in option**: Thus, financial and non-financial default triggers give lenders an option to "step in", which in turn, should impact the debt value. In case of a breach of a debt covenant, debt owners have the right to get involved in the management of the project company. In this context, debt holders can put in a "cash sweep" to

2. Characteristics of Private Infrastructure Debt

accelerate debt payments, or reschedule debt payments to gain more financial flexibility to resolve outstanding issues.

- **Cash sweeps**: cash sweeps are a form of compulsory prepayment. After an agreed-upon distribution to shareholders, which can be zero, any remaining CFADS balance is used to prepay the debt. This is to minimise the effects of substantial fluctuations in cash flows on credit risk and to use the excess cash generated in good periods to reduce outstanding debt and protect against periods with lower than expected cash flows. Of course cash sweeps introduce prepayment risk for investors.

- **Cash clawback**: Under a clawback provision, equity investors agree that if problems occur with future cash flows, they will repay or lend to the project company up to the amount they have received in dividends or other distributions over a set period of time.

- **Reserve accounts**: Reserve accounts are established to reserve cash during periods of higher earnings to service debt payments during periods of lower earnings. These accounts provide security for lenders against short-term cash problems, and can also be set up to fund future expenditures. Reserve accounts may also segregate funds based on their use. For example, debt service reserve account (DSRA) contains funds to service next period's debt payment (principal + interest), and tax reserve accounts contain funds to pay tax liabilities that have been incurred but would be paid in the future.

- **Prepayment option**: PF loans often allow prepayments at little to no cost. If the project does well, the SPE may take advantage of this prepayment option to refinance at lower rates. However, with the development of institutional investors' involvement in lending to infrastructure projects, prepayment may become more penalising for SPEs since these investors tend to be looking for instruments with a known duration.

At this stage, we note that traditional capital budgeting methods fail to take into account the effects of these covenants and embedded options. Structural credit models, on the other hand, can incorporate the effects of these covenants through their effect on the cash flows to debt holders.

In what follows, we discuss in more details three important points which must inform our approach to valuing infrastructure project debt, namely, the identification of the default point, the role of debt schedule re-organisation and the impact of illiquidity.

2.3 Identifying Default Triggers

Default mechanisms in project finance have two important dimensions: first, the default point is more straightforwardly known than in standard corporate finance; and second, the presence of debt covenants that impose other obligations on the borrower in addition to the debt repayment means that

2. Characteristics of Private Infrastructure Debt

technical defaults are a prevalent form of credit event.

In structural models of standard corporate debt, default is generally modelled as crossing a threshold point below which the total value of the firm's assets is less than its short and medium term liabilities. This is because as long as the total value of the firm is higher than its near term liabilities, equity holders can raise more cash by issuing new equity or debt, and satisfy their current debt obligations. For PF SPEs, this is not the case because equity holders are constrained in their ability to raise more cash by issuing new debt and equity to preserve the value of existing debt holders' security (see Yescombe, 2002, sections 13.7 and 13.10). The non-recourse nature of the equity investment and the inability of the firm to increase its borrowing make default easier to predict than in standard corporate finance.

In project finance, the relationship between the firm's free cash flow or cash flow available for debt service (CFADS) and the expected senior debt service i.e. the ability of a given SPE to service its senior debt obligation, is captured by a **debt service cover ratio** (DSCR), which is routinely monitored by project finance lenders for each SPE. The DSCR at time *t* is written:

$$DSCR_t = \frac{\text{CFADS}_t}{\text{DS}_t^{\text{BC}}} \tag{2.1}$$

in each period t=1,2,..T for a project financing of maturity T; DS^{BC} is the base case debt service.

We note that as a function of the CFADS i.e. the underlying process explaining firm value, **the distribution of the DSCR at time** t **(**$DSCR_t$**) in project finance can capture both expected asset values and volatility**.

Moreover, the DSCR provides an *unambiguous* definition of default.

Thus, a "hard" default of the SPE i.e. an actual default of payment, can be defined in terms of the *ex post* CFADS at time t, as:

$$\text{Default}_t \iff \text{CFADS}_t < \text{DS}_t^{\text{BC}} \tag{2.2}$$

which can be expressed in terms of *ex post* $DSCR_t$ as:

$$\text{Default}_t \iff DSCR_t = \frac{\text{CFADS}_t}{\text{DS}_t^{\text{BC}}} < 1 \tag{2.3}$$

By definition, if $DSCR_t$ equals unity, the SPE is just able to service its senior debt during the relevant period, and if it falls below unity, the borrower can *unambiguously* be considered in default.[8]

Credit events may also be defined more loosely. For example, in the Basel-II framework, project finance default is defined as '*...past-due more than 90 days on any material credit obligation to the banking group*' (BIS, 2005).

Thus, unlike standard corporate debt where covenants typically only relate to the financial state of the firm, project finance SPEs can also experience *soft* or *technical* defaults e.g. a low *ex post DSCR* may constitute a breach of the loan's covenants and also be considered an event of default.

8 - Moody's definition of project finance default as '*a missed or delayed disbursement of interest and/or principal...*' Moody's (2013) is congruent with this view.

2. Characteristics of Private Infrastructure Debt

The formulation of the default point above suggests that *credit events can be very finely defined* in project finance, and that lenders may consider an SPE to be in default and take remedial action long before it has become unable to repay its debt.

Ex ante i.e. at the time of financial close, lenders typically require the DSCR to be significantly higher than unity in order to create a credit risk buffer and also so that equity/junior distributions can be made once senior debt obligations have been met. *Ex post*, if $DSCR_t$ is too low, it can trigger one of the covenants described above.

Hence, the default point in project finance at time t can be defined as:

$$DSCR_t = 1.x \text{ with } x \geq 0$$

And since the DSCR can provide an unambiguous default point of infrastructure project finance debt, its probability of default at time t can be written:

$$p_t = \Pr(DSCR_t < 1.x | min_{j<t} DSCR_j \geq 1.x)$$

i.e. it is the probability that the DSCR reaches the default point, conditional on there having been no default until that time.

Hence, knowledge of the distribution of $DSCR_t$ for a category of project financing and of the DSCR-related covenants of a given loan is sufficient to identify and predict default in project finance. In chapter 3, we also show that knowledge of the first two moments of the distribution of $DSCR_t$ is sufficient to derive the SPE's distance to default, which is instrumental in our valuation model.

2.4 Reorganisations

Reorganisations are the result of the embedded options discussed above. We use the term reorganisation to refer to any change in the PF SPE's capital structure or debt service schedule (face value, maturity, and seniority) from the base case scenario. Such reorganisations are very common in project finance (see Yescombe, 2002, sections 7.7 and 13.6).

SPEs reorganise both during financial distress (to avoid bankruptcy), and when the firm's free cash flow is sufficiently high (to take advantage of a lower credit risk to refinance at lower rates).[9]

Such reorganisations can change the SPE's debt service schedule and hence its default threshold can also deviate from the base case scenario.

Crucially, in the case of reorganisations triggered by financial distress, the period between the maturity of the debt and the maturity of the project, which is often referred to as *the tail of the loan*, can allow for debt service re-organisations that leave the value of the initial debt quasi- or completely intact. It allows debt holders to restructure debt schedule and recover any losses suffered during the original maturity of the debt (see Yescombe, 2002, sections 12.9.4 and 13.2).

9 - The suitability of refinancing depends on the maturity of existing debt, debt covenants that may penalise refinancing, and external market conditions. If the debt covenants allow refinancing at little to no cost, as is usually the case with bank loans (see Yescombe, 2002, section 13.6), and demand for infrastructure projects is high, an SPE may be able to refinance at lower costs. However, if the existing debt is expiring soon and the demand for infrastructure debt is low, refinancing can be costly. Long-term investors' greater aversion to refinancing, which can significantly reduce the duration of their fixed income portfolio may lead to the more frequent introduction of prepayment charges and to fewer refinancings.

2. Characteristics of Private Infrastructure Debt

Hence, both the recovery rates and default threshold are endogenous in project finance, and a PF debt valuation model must incorporate this endogeneity.

2.5 Illiquidity and Lumpiness

Finally, our project finance debt valuation framework must integrate certain contingent features of these instruments, namely illiquidity and size.

Several years of project preparation can go by before PF debt is originated. It is then typically held to maturity by lenders or investors and trades very infrequently. Moreover, even secondary market transactions require significant due diligence and documentation, such that transaction costs remain significant, hampering liquidity.

A direct consequence of this illiquidity is the significant transaction costs associated with buying or selling such instruments, and the absence of time series of market prices for PF debt. The presence of transaction costs makes models built on the assumption of frictionless markets unsuitable for PF debt. The lack of market prices makes so-called reduced form models of credit risk, which rely on observed market prices, unsuitable for pricing PF debt. Such models could be employed if *comparable* traded debt securities existed but because of the many idiosyncratic features of PF loans, this is unlikely to be the case. In fact, it is our premise that it is not the case.

The lot size problem familiar to investors in real estate also befalls infrastructure project finance debt portfolios. Project finance investments usually have long maturities and require large amounts of capital. At a given point in time, there may not be a large number of projects available for financing. Hence, it may not be possible for investors to access a sufficiently large number of projects at time t to attain their desired level of diversification.

Hence, even the risks that are idiosyncratic in theory may remain un-diversifiable and would have to be priced if they are correlated with the investors' portfolio (i.e. their marginal contribution to the portfolio is non-negligible). Similarly some of the otherwise diversifiable risks may not be so due to the long horizon of infrastructure projects. For example, inflation risk and currency risk can be hedged using inflation indexed bonds and currency swaps. But bonds and swaps with sufficiently long maturities may not be available.

Markets for unlisted infrastructure debt thus tend to be both incomplete and not frictionless because of these instruments' illiquidity and lumpiness. This is likely to lead to divergent investor valuations determined in part by risk preferences and by the size of the infrastructure debt allocation in their respective portfolios. Hence, a valuation model of unlisted infrastructure loans must incorporate the existence of bounds on value i.e. the absence of a single market price for a given instrument.

Due to these unique characteristics, corporate debt valuation models cannot be directly applied to the PF debt. In the

2. Characteristics of Private Infrastructure Debt

next two chapters, we review the existing literature on debt valuation, and propose a valuation methodology that takes into account these PF debt characteristics.

2.6 Documented Performance

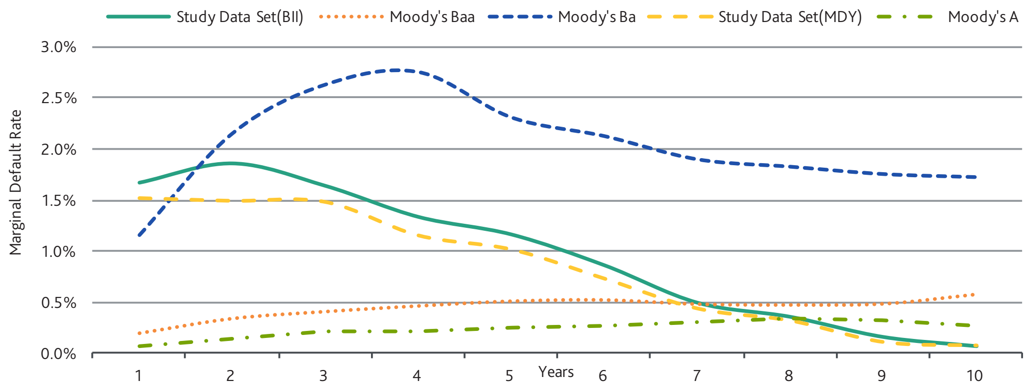
Little empirical knowledge exists of the performance of project finance debt but several studies have been conducted by rating agencies focusing on the likelihood of default. Such studies include Moody's (2012, 2013, 2014) as well as Standard and Poor's (2013) and propose to measure the number of defaults observed within a population of loans at a given point in time and in each loan's lifecycle. Several stylised facts are frequently abstracted from these reports:

- On average the available sample of project finance loans exhibits marginally decreasing cumulative default rates in time, which is the result of a decreasing annual probability of default as project loans mature;
- As a consequence, the available sample of project finance loans exhibits a continuous credit risk transition over a period of approximately ten years, from a triple-B equivalent to a single-A equivalent;
- As shown on figure 7, the observed probability of default in Moody's sample (green line) trends form around 2% around the time of financial close to near zero after ten years;
- The same study also attempts to isolate so-called public-private partnership (PPP) projects that mostly receive a contracted income stream from the public sector and finds that their probability of default average 0.5% across their entire lifecycle.

These results, may be affected by sampling biases (see Blanc-Brude and Ismail, 2013) and lack any clear identification of the relationship between credit risk and the underlying projects' business model. Nevertheless, they are informative and provide us with an empirical point of comparison to the output of our model, which we discuss in the next section.

2. Characteristics of Private Infrastructure Debt

Figure 7: Marginal probability of default in a sample of project finance loans



Source: Moody's (2013)

3. Approaching the Valuation of Infrastructure Debt

3. Approaching the Valuation of Infrastructure Debt

In this chapter, we discuss our approach to value infrastructure project finance loans.

Models commonly employed to value PF loans include capital budgeting models that determine a project's feasibility using NPV, IRR, payback period etc. under a base case scenario. These models are, however, static in nature, ignore the effects of debt covenants and embedded options, and fail to shed any light on the evolution of the credit risk profile in time or the opportunity to recoup losses "in the tail". Such models often assume a constant loss given default, and a constant discount rate. Both these variables, however, should depend on the underlying risk profile of the project, and should change with time as the risk profile changes.

Multinomial tree-based option pricing models have also been applied to PF debt (see for example Ho and Liu, 2002; Wibowo, 2009) and can take into account some debt covenants, but fail to incorporate the endogenous nature of credit risk, which makes PF debt cash flows path dependant.

In addition, these models often use the SPE's WACC (Wibowo, 2009) or CAPM-based discount factors to discount future cash flows. While the SPE's WACC is certainly relevant in determining the feasibility of the project from the perspective of the SPE, it is not relevant for valuation from the perspective of outside investors, whose WACC may be very different from the SPE's, and may not matter in their decision to invest in PF debt at all.

A more sophisticated approach used by banks to measure the risk based performance of loans is the Risk Adjusted Return on Capital or RAROC: the ratio of the adjusted income from the loan to a risk-based capital requirement, that is, the amount of capital needed to limit total default probability to a certain level, weighted by the marginal contribution of the loan to total loss for the bank. The decision to lend is made if the RAROC exceeds the bank's net cost of capital or hurdle rate (see Shearer and Forest Jr, 1997; Froot and Stein, 1998; Aguais et al., 2000).

This approach has several limitations, chief amongst which is the use of the bank's internal cost of capital to determine required rates of return. The RAROC measure is also insensitive to the structure of the security (type of loan, amortisation of principal, covenants, collateral requirements, repayment rights, pricing grids, etc) and it requires the same discount rate for an instrument with embedded call option and associated pricing grids than it does for one with no embedded options.

Hence, existing loan valuations approaches can be described as inadequate for the purposes of long-term investors who need performance measures relevant to risk management, hedging and portfolio management. In what follows, we briefly review the literature on asset pricing and credit risk modelling, and discuss what theoretical framework is most adequate to design a valuation framework of private infrastructure project finance debt.

3. Approaching the Valuation of Infrastructure Debt

3.1 Literature Review

We first briefly describe different approaches to asset pricing and introduce the notion of market incompleteness and the absence of a unique pricing measure. We then discuss credit risk models used to value corporate securities and how they may apply to project finance loans.

3.1.1 Asset Pricing Models

Equilibrium Pricing

Equilibrium pricing models determine asset prices such that the supply of every asset equals its demand, as should be the case in equilibrium. They are very general and can be used to value any security, given its cash flow distribution. Their main strength is to relate security prices to the their fundamental determinants: investors' risk preferences, endowments and the distribution of cash flows. Modelling investors' preferences, however, is not an easy task, and the outcome of such models can be very sensitive to the assumptions made in this regard.

As a consequence, they are rarely used by practitioners. Examples include utility based indifference pricing models that model investors' utility from consumption, leisure etc. to derive their risk preferences. The price of a risky security is then obtained as the price that makes investor's utility from investing in the risky asset equal to its utility from investing in the risk-free asset. That is, the fair price of a security for an investor is the price that leaves investor indifferent between investing in the risky and the risk-free asset (at a different unit price).

A simple example of the utility based pricing models is the Capital Asset Pricing Model (CAPM), which can be obtained as an outcome of expected utility maximisation when agents have quadratic utility functions (Berk, 1997). [10]

A second class of equilibrium pricing models incorporates investors' risk preferences through the subjective probabilities that investors assign to risky cash flows. These probabilities are known as the *risk neutral* probabilities, and the valuation method is called risk neutral valuation, as the investor behaves as a risk neutral investor would under the equivalent probability measure (i.e. discounts future cash flows at the risk-free rate).

In practice, risk-neutral valuation adjusts for risk aversion by assigning a lower probability to riskier cash flows, and hence decreasing their expected value under the risk neutral distribution, instead of discounting the expected values under the physical distribution at a higher discount rate — both approaches are equivalent. Examples of such models can be found in Wang (2002) and Madan and Unal (2004).

Relative Valuation

Relative valuation models determine asset prices relative to other assets. This approach relies on arbitrage principle, also knows as the law of one price. Arbitrage principle requires the market prices of different securities to be consistent. That is, the prices of two securities with identical cash flows should be the same. If the prices of two securities with identical cash flows are not

10 - More advanced examples of utility based pricing models can be found in Musiela and Zariphopoulou (2004), and Jong (2008), and Henderson and Hobson (2004) provides a review of such models.

3. Approaching the Valuation of Infrastructure Debt

equal, an investor can construct an arbitrage – an opportunity to earn risk-less profit – by taking a short position in the over-priced security, and a long position in the under-priced security. This approach dates back to Modigliani and Miller who used arbitrage arguments to establish irrelevance of firm's capital structure (see Modigliani and Miller, 1958) and relies on observing market prices for some assets, and under-standing the riskiness of assets relative to each other to determine the relative prices.

Relative valuation is less general than equilibrium valuation, as does not say how other assets should be priced. That is, it takes the prices of other assets as given, and does not seek to determine if they are fairly priced. Moreover, it requires *market completeness*, which stipulates that the cash flows of each security can be replicated using other securities, to determine a unique price for the security being valued.

This approach, however, is less subjective than the aforementioned equilibrium pricing models, as it does not require investors to specify their personal risk preferences. In other words, all investors who prefer more wealth to less would take an arbitrage opportunity, irrespective of their current endowments and risk preferences, and the market should quickly reach an arbitrage-free equilibrium. We return to this very commonly used valuation methodology in section 3.1.2.

Incomplete Markets

Both equilibrium and relative valuation models typically assume that markets are complete and frictionless,[11] which is not the case for private and illiquid project finance debt.

Market incompleteness refers to the situation when the cash flows of a security cannot be perfectly replicated using traded securities. In incomplete markets, relative valuation models cannot be used to obtain a unique price for a traded security. The reason is that if the cash flows of a security are not "spanned" by other securities, then one cannot construct a replicating portfolio, and the arbitrage principle cannot be used to arrive at a unique pricing measure for the security being valued.

In short, if markets are not complete, one cannot always uniquely specify the price of one asset relative to other assets, that all investors would agree on, irrespective of their individual endowments and risk preferences. In incomplete markets, the arbitrage principle only leads to *upper and lower bounds* on the security prices. That is, the arbitrage principle can tell us that the actual price of the security will lie within a "reasonable" range, but cannot tell us what the required price for individual investors is.

With incomplete markets and transaction costs, investors cannot always arbitrage even when the same security trades at different prices. Market frictions are significant in project finance debt, and observed prices *for the same security* are thus likely to lie within a range determined by arbitrage *bounds*.

11 - In frictionless markets there are no transaction costs and no constraints on short selling.

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Market incompleteness can be incorporated in valuation models using *minimum martingale measures* or so-called *approximate arbitrage*.

Minimum Martingale Measures

Minimum Entropy Martingale Measures (MEMMs) and Minimum Variance Martingale Measures (MVMMs) belong to equilibrium pricing methods that use probability transforms to incorporate investors' preferences. MEMM methodologies obtain the risk-adjusted probability density by minimising its entropy relative to a prior probability density function (often the physical density function), and is equivalent to the maximisation of expected utility of terminal wealth. MVMM methods seek to minimise the variance of the martingale measures. This approach leads to a unique price for securities, and hence is only useful when markets are frictionless. For detailed discussion of MEMMs and MVMMs (see Frittelli, 1995, 2000).

Approximate Arbitrage

These models use the equilibrium pricing techniques, but instead of making strong assumptions about investors' preferences to arrive at unique prices for the assets, they make weaker assumptions about investors' preferences to strengthen the arbitrage bounds. That is, approximate arbitrage models are between no-arbitrage models that make very weak assumptions about investors' preferences and result in very wide bounds on asset prices, and equilibrium pricing models that make strong assumptions about investors' preferences and lead to unique asset prices.

Hence, the resulting price bounds are stronger than in no-arbitrage models, but weaker than in equilibrium pricing models (see for example Cochrane and Saa-Requejo, 2000; Bernardo and Ledoit, 2000; Carr et al., 2001). Such models allow for the existence of frictions or transaction costs which, assuming they do not prevent all trades, can be incorporated in the price required by investors.

The idea behind approximate arbitrage is very intuitive: Carr et al. (2001) discusses "acceptable" prices and Cochrane and Saa-Requejo (2000) writes about "good deal" bounds i.e. despite the absence of a unique pricing measure, there are prices beyond which no investors can go either because they imply close to no risk aversion (upper price bound) or would allow an arbitrageur to make a sizeable profit at very little risk even in the presence of large transaction costs (lower price bound).

Since we want to develop a measure of the 'market value' of infrastructure debt but illiquidity and transaction costs imply the absence of a unique pricing measure, **we conclude that relative pricing combined with approximate arbitrage modelling provides the most promising theoretical framework for the valuation of private infrastructure project finance debt securities.** Next, we discuss the use of credit risk models applied to corporate debt and how they may be used in the case of infrastructure project debt.

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3.1.2 Corporate Debt Valuation Models

Corporate securities are mostly valued using relative valuation models. In this section, we discuss the two main classifications of relative valuation models: 1) reduced form models and 2) structural models.

Reduced Form

Reduced form models specify the probability of default and loss given default exogenously, allow these quantities to depend on external state variables, and make the following assumptions (see Jarrow and Deventer, 2013)

1. Some of the firm's debt trades in frictionless, arbitrage free markets;
2. The state of the economy can be described by a vector of stochastic variables X_t, such as risk-free rate, rate of inflation, unemployment rate, GDP etc;
3. The probability of default can be modelled as a *Cox* process with default intensity $\lambda(X_t)$, and the probability of default over a time interval $[t, t+\delta]$ is given by $\lambda(X_t)\delta$;
4. The default of any given company in a given state of economy is a random event and the firm's credit risk is idiosyncratic; and
5. The percentage loss given default (LGD) is $0 \leq l(X_t) \leq 1$.

The main advantages of reduced form models are that since the inputs (PD and LGD) are observable, credit risk measures can depend on the state of economy, and do not require a specification of the company's balance sheet structure.

The main limitation of reduced form models is that market data is necessary to calibrate them, and that price staleness can affect model accuracy.

We note that existing empirical work on project finance debt (see for example Moody's, 2014) takes an implicit reduced form approach to credit risk measurement since it relies solely on observing discreet events of default and recovery. Nevertheless, even if existing samples are large, [12] they are not without biases, as we show in Blanc-Brude and Ismail (2013) and current datasets do not allow any research into the statistical determinants of credit events observed in project finance.

12 - In the case of recovery rates, Moody's (2014)'s sample is not large.

Reduced form models' heavy reliance on market data make them a poor choice for project finance debt valuation, for which not enough data can be available to calibrate exogenously specified default and recovery processes. In fact, the illiquidity of these instruments guarantees that *there will never be enough data in the cross-section and time series to apply a reduced form model*. Instead, we argue that structural models, which we describe below, are a superior choice.

Structural Models

Unlike reduced form models that specify a default process exogenously, structural models postulate the existence of a default triggering mechanism i.e. a discrete event at the threshold between two states (default vs. no default), the probability of which is determined endogenously. In other words,

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default events do not occur randomly but are linked to firm's assets and liabilities.

In the classic Merton (1974) model of valuation of corporate debt, the value of the company follows a stochastic process (V_t). The company is financed from debt and equity, its debt is a single obligation and resembles a zero coupon bond with face value B and maturity T. At time t, the value of the firm is the sum of its equity S_t and debt B_t ($V_t = B_t + S_t$, for $0 < t < T$).

In this model, the firm does not pay any dividends nor issues any new debt. If at maturity the value of the firm is less than its liabilities ($V_T < B$), the firm is considered to be in default. The equity holders then choose not to provide any new equity capital as an expression of their 'limited liability option' and hand over the firm to the debt holder, which liquidates the remaining assets and receive the proceeds $B_T = V_T$. If there is no default, the debt holder receives the payoff B, and equity holders receive the remaining of the firms value $V_T - B$.

It is now a classic result that this model implies for the value of the firm's equity at time T to be equivalent to the payoff of a European call option on V_T, while the debt value equals the nominal value of liabilities (as risk-free zero coupon bond) less the payoff of a European put option on V_T. Under a number of assumptions, there is a closed form solution for the value of the firm's debt, which can be priced as the value of standard plain vanilla options (see McNeil et al., 2005).

Of course, the original Merton model has been criticised for making a number of assumptions, including the lognormal distributions of returns and a simplistic capital structure (the firm borrows once and subsequently de-leverages). The definition of default used in the Merton model has also been criticised: the default point is not only assumed to be known unambiguously (when asset value falls below liabilities) but the firm *must* default exactly when this point is reached, neither of which is self-evident empirically. However, it should be clear from the discussion in chapter 2 that the Merton model is rather well-suited to project financing: SPEs borrow once and subsequently deleverage, default is unambiguously known and actively monitored, and the underlying process driving asset value (CFADS) can be captured by the distribution of the DSCR.

Furthermore structural models have been further developed to extend the original Merton model and address most of the issues found in the asset pricing literature. These include models that incorporate complex capital structures,[13] stochastic interest rates,[14] stochastic volatility,[15] jump diffusion processes,[16] incomplete information,[17] exogenous[18] and endogenous[19] default thresholds, and strategic debt service.[20]

Structural models are thus the most suitable choice for PF debt, as the primary input of these models is the value of the firm's assets and loan covenants, which we know can be observed in the case of PF

13 - Jones et al. (1985), Jones et al. (1984) and Black and Cox (1976))

14 - Longstaff and Schwartz (1995), Heston (1993) and Shimko et al. (1993)

15 - Heston (1993) and Guo et al. (2009)

16 - Delianedis and Geske (2001) and Zhou (1997)

17 - Bellalah (2001) Duffie and Lando (2001), Guo et al. (2009), Giesecke (2004) and Giesecke (2006)

18 - Black and Cox (1976) and Longstaff and Schwartz (1995)

19 - Leland (1994) and Anderson and Sundaresan (1996)

20 - Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997)

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SPEs, and because they can incorporate the endogenous credit risk dimension.

3.1.3 Conclusion

Existing loan valuation methodologies used by practitioners have numerous shortcomings: they often do not take into account the dynamic risk profile of project finance loans, can rely heavily on historical data. Nor do they derive credit risk variables from the firm's fundamentals, but rather make *ad hoc* assumptions about loss given default, recovery rates etc. Finally they tend to use the loan's interest rate or the cost of capital of the bank to discount future cash flows, and there is no reason to believe *a priori* that this is the correct rate of discount for other investors. Thus, current loan valuation methodologies are unsuitable to value private infrastructure project debt as an illiquid instrument for long-term investors.

Corporate debt valuation methodologies are more adequate but also rely heavily on market prices and assume complete and frictionless markets. While these assumptions can be justified for corporate bonds, which often trade in liquid markets, they are not realistic for project finance loans, which are typically illiquid and do not satisfy the requirements of market completeness. This is the primary reason why these methodologies have not been employed in loan valuation, despite being heavily used for the valuation of corporate securities.

A project finance loan valuation methodology needs to take into account illiquidity and market incompleteness, but also the endogenous dimension of credit risk, the presence of debt covenants which create extensive control rights for lenders in certain states of the world, as well as high transaction costs.

Structural models are flexible enough to accommodate all these characteristics but existing structural models are not geared to take into account the unique features of project finance loans.

Our proposed model, which we outline in the next section, extends existing structural models to take into account these features, and makes the following contributions:

- It uses observable inputs, and hence can be calibrated and updated as more data becomes available;
- It integrates the extent of market incompleteness into the model, without sacrificing the benefits of using market data where available;
- It takes into account the endogenous nature of loan cash flows and credit risk in project finance;
- It distinguishes between technical and hard defaults, and hence can value the lenders' step-in rights;
- It can take into account common project finance debt covenants such as reserve accounts, cash sweeps, clawback provisions etc;
- It can take into account the unique circumstances of investors, which could stem from their regulatory requirements, the nature of their liabilities, and their level of diversification, when valuing infrastructure project loans.

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3.2 Proposed Approach

3.2.1 Intuition

Our objective is to design a valuation model which takes into account the characteristics of infrastructure project finance debt described in chapter 2 and builds on existing asset pricing theory, in particular structural models of credit risk and the requirement to have multiple prices for the same instruments (market incompleteness). Finally, we aim to minimise data collection requirements to make this methodology as easy to implement as possible for investors and practitioners.

Our model rests on the following intuition: If $CFADS_t$ is the underlying stochastic process explaining the SPE's total value, and since lending to a project company can usefully be described as the equivalent of writing a derivative contract on the project's CFADS with the *ex ante* agreed debt service as the strike price, then the dynamics of the debt service cover ratio (DSCR) in project finance, which is routinely monitored by lenders, provide us with unique insights into the value of the firm, because the $DSCR_t$ is a reflection of the underlying free cash flow, the firm's financial structure (effective leverage) and the default threshold. It follows that the knowledge of $DSCR_t$ along with debt covenants and the size of the loan's tail are sufficient to value the firm and its debt.

Our valuation framework thus consists of two main components:

1. A model of the free cash flow of the SPE and to debt holders in all states of the world, which has two sub-components:
   a) We first build a model of $CFADS_t$ using DSCR dynamics and the base case debt schedule. In this paper, we use *a priori* parameter values for the distribution of $DSCR_t$ based on our knowledge of project finance debt structuring. In a forthcoming paper (Blanc-Brude and Hasan, 2014), we describe a Bayesian approach to update this knowledge of the distribution of $DSCR_t$ as more data is collected using the data collection template defined at the end of this paper.
   b) Next, to model changes in the debt schedule following any credit event, we take a game theoretic approach to determine the outcome of negotiation between debt and equity holders with both parties acting in their self-interest. The relative bargaining powers of parties are determined by their contractual rights and obligations, as well as the value of their outside options.

2. A valuation and risk measurement model for a given debt schedule with a given set of debt covenants: we use a structural model to derive the risk return characteristics using these cash flow and debt renegotiation models described above for a range of subjective risk preferences representing the approximate arbitrage bounds on value.

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Next, we outline each step of the valuation framework, from modelling cash flow dynamics, to debt renegotiation, to discounting cash flow projections.

3.2.2 Cash Flow Dynamics

The first step in our valuation framework is to project the cash flow available for debt service (CFADS) to the SPE in every state of the world. As argued earlier, in non-recourse project financing, the discounted CFADS is equivalent to the firm's asset value.

Cash flows to debt holders are then determined by the combination of the base case debt service and the realised CFADS. A hierarchical payment structure known as a *cash flow waterfall* is typically ordered thus:

1. Senior debt service (principal + interest)
2. Payments to reserve accounts
3. Cash sweeps
4. Debt prepayments
5. Subordinated debt service
6. Distributions to equity holders

That is, the CFADS is first used to make the scheduled debt payments, and then to satisfy any debt covenants related to reserve accounts and cash sweeps. The remaining cash can then be used prepay some or all of the existing debt, and to make payments to equity holders.

The Role of $DSCR_t$

While it is part of the *ex ante* cash flow modelling, project CFADS is not necessarily known or monitored *ex post*. However, this measure can be inferred from the debt service cover ratio (DSCR).

Given the definition of $DSCR_t$ given in equation 2.1, the CFADS for a given period is simply obtained as:

$$\text{CFADS} = \text{DSCR}_t \times \text{DS}_t^{\text{BC}} \tag{3.1}$$

with DS_t^{BC}, the debt service. The same relationship holds in expectation.

In other words, as long as the base case debt service is known, we can reduce the question of modelling the free cash flow of the firm in project finance to that of the dynamics of $DSCR_t$ and its determinants.

DSCR Families

With significant data paucity in time series and in the cross-section of projects, as discussed in chapter 1 and in Blanc-Brude (2014), we cannot hope to observe sufficiently large and representative sample of DSCR observations to determine the characteristics of DSCR dynamics empirically. Instead, we must make *a priori* choices about sub-groups of project financial structures, which we expect to correspond to reasonably *homogenous* DSCR dynamics.

In other words, our objective is to partition the infrastructure project finance universe into a parsimonious set of tractable cash flow models, which can be calibrated using available data in due course. As discussed above, part of the objectives of this paper is to define exactly what data must be collected for this purpose.

As is also identified in Blanc-Brude (2014), reasonably homogenous groups of instruments can then be used as the *building blocks* thanks to which the systematic

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performance of different exposures to infrastructure debt can be identified, and later portfolio (benchmark) construction can take place.

In this paper, we identify two generic groups of infrastructure project finance structures, each of which represents an ideal type corresponding to numerous existing projects (but not all).

At financial close, numerous infrastructure projects are typically structured either with a rising or a flat base case DSCR profile.

A rising DSCR profile exhibits both a rising mean and an increasing volatility of $DSCR_t$. That is, creditors demand a higher DSCR in the future to protect themselves against rising expected volatility of CFADS. Such projects also have longer tails and exhibit between 70% and 80% of initial senior leverage. Projects that are exposed to market risk, such as a power plant that sells electricity at market prices, are structured to have a rising DSCR profile. We refer to these projects as *Merchant* infrastructure.

Conversely, a flat DSCR profile has a constant mean and implies constant expected cash flow volatility. Projects with little to no market risk are structured with a flat DSCR. They also have shorter tails and a higher level of senior leverage usually around 90%. Moreover, contrary to projects with a rising DSCR, which effectively de-leverage as their lifecycle unfolds, projects with a constant DSCR stay highly leveraged until the end of the debt's life (otherwise their DSCR would rise). Examples of these projects include social infrastructure projects, such as schools or hospitals that receive a fixed payment from the public sector. We refer to these projects as *Contracted* infrastructure.

We note that other generic models of project finance structures can be described, not least a hybrid version of the two cases discussed above. However, the Merchant and Contracted cases provide a sufficiently rich set to illustrate our methodology.

3.2.3 Debt Restructuring

Identifying DSCR dynamics leads to a $CFADS_t$ model conditional on the base case debt schedule. However, as discussed in section 2, the base case debt schedule can itself change upon the reorganisation that can follow a credit event. The resulting change in the debt schedule changes the expected DSCR profile post-reorganisation. To capture expected this change in the DSCR profile, we need to model the change in debt schedule itself.

To model debt service reorganisations (or restructurings) upon default, we first assume that the equity holders honour their debt obligations as long as there is sufficient free cash flow (CFADS) available to make the scheduled debt payment and do not engage in so-called strategic debt service.[21]

Crucially, we distinguish between the outcome of so-called *technical* and *hard* defaults i.e. credit events triggered by a breach of covenant such as a low DSCR, or actual defaults of payment. In what

21 - We could have assumed that the equity holders choose to pay less than the scheduled debt payment even if there is sufficient cash available. We ignore such "strategic debt service" because in project finance the funds for non-operating expenses are held under joint control with the lender's agent (see Yescombe, 2002, section 13.5). Equity holders are not likely to have enough control on free cash flows to pursue strategic debt service.

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follows, we detail each case in turn, taking into account the control rights ad relative bargaining power of lenders in each situation.

Restructuring upon a technical default
Covenant breach or technical default gives debt holders the right to step-in, impose certain management decisions, and require the restructuring of the outstanding debt. In practice, equity investors may also be required to inject more capital in the project company, but we ignore this possibility and assume that debt is only paid with the free cash flows of the SPV.

In a situation of technical default, **lenders can aim to maximise the value of the restructured debt service relative to the original outstanding debt amount**,[22] but not more.

22 - The outstanding debt at any point is simply the amortised value of the debt, which can be obtained by discounting the remaining scheduled debt payments by discounting them at the internal rate of return.

Indeed, the project company does not go into bankruptcy and equity holders continue the construction and/or operation of the project (see Gatti, 2013, section 7.2.3.11.2 on negative covenants). They do not exercise their limited liability option and retain significant control rights as owners of the project company.

We further assume that debt holders will have to incur some *restructuring costs* to have the debt reschedules. Therefore, they only choose to reschedule the outstanding debt if they can impose a new debt schedule such that the *market value* of the new debt, net of restructuring costs, is higher than the market value of the existing debt.

Thus, restructuring PF debt upon a technical default involves the following steps.

1. Determine the *outstanding debt value*: the present value of the existing debt schedule discounted at the original IRR of the loan;
2. Determine the *market value* of the existing debt schedule i.e. the risk-adjusted value of debt discounted at the appropriate rate, which is likely to be different from the original IRR. We propose determine the market value of the debt using a risk neutral valuation model as discussed in section 3.2.4;
3. Pick a new debt schedule such that its value when discounted at the original IRR of the loan is the same as the original outstanding debt value;
4. Determine the market value of this new debt schedule;
5. If the market value of the new debt schedule, net of rescheduling costs, exceeds the market value of the original debt schedule, the new debt schedule is preferred;
6. These steps can be repeated until a debt schedule has been found that maximises the market value of the restructured debt, for example by minimising credit risk and extending the debt service in the "tail" of the loan.

Thus, a situation of technical default gives lenders **control rights that allow them to maximise their expected recovery rate**. Technical defaults are the most frequent type of credit event in project finance for the obvious reason that the CFADS is more likely to reach some threshold set before a

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hard default can occur, than to actually lead to a default of payment.

In chapter 5, we report both technical and hard defaults (using Moody's definition in the latter case), and technical defaults are by far the most frequent.

Restructuring upon a hard default

Hard defaults create a more complex set of outcomes. We treat a hard default as an event upon which the existing contract between debt and equity holders is impaired, and equity holders loose the control rights of the SPE, which is the result of their original *share pledge*. However, because equity holders can now exercise their limited liability option, *depending on the costs to lenders implied by an actual take-over of the SPE*, the original equity holders have not lost all bargaining power.

After a hard default, lenders have the control of the SPE and they can aim to maximise the value of these control rights. Their preferred course of action may or may not involve the original equity owners.

Next, we describe the conditions under which renegotiations can take place upon a hard default and their possible outcomes in the debt renegotiation model.

Decision to exit or renegotiate

We assume two possible outcomes upon a hard default: *exit* or *renegotiation*.

By "exit" we mean that lenders proceed to either:

- Enter into a new contract with a new set of equity investors;
- Sell the loan in the distressed debt market;
- File for bankruptcy or sell the SPE.

Thus, while the SPE may well continue to exist, the original equity investors are forced out and effectively loose the value of their investment at that time.

By "renegotiation", we mean that the original debt and equity holders manage to restructure the SPE in a mutually advantageous manner, and agree to enter into a new contract. In fact, empirical studies on project finance suggest that this `work-out' scenario is the most common upon a hard default in project finance (Moody's, 2014, see for example).

However, since lenders are effectively in control of the SPE, we can assume that they will choose the course of action that first maximises their own value, which may or may not leave a share of expected cash flows for the current equity owners.

We argue that, with rational players, debt renegotiation (the non-exit scenario) after an event of hard default can only occur if the following three conditions are satisfied:

(C_1) Both debt and equity holders can gain at least as much from renegotiation as they would upon exit;

(C_2) At least one of the stakeholders can get more than what they would if nothing was done;

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Figure 8: Renegotiation and exit values immediately following a hard default at time τ. V^{reneg} and V^{exit} represent the value of the SPE under the renegotiation and exit scenarii, respectively; c^{reneg} and c^{exit} represent renegotiation and exit costs, respectively; NPV^{reneg} and NPV^{exit} represent renegotiation and exit values, respectively.

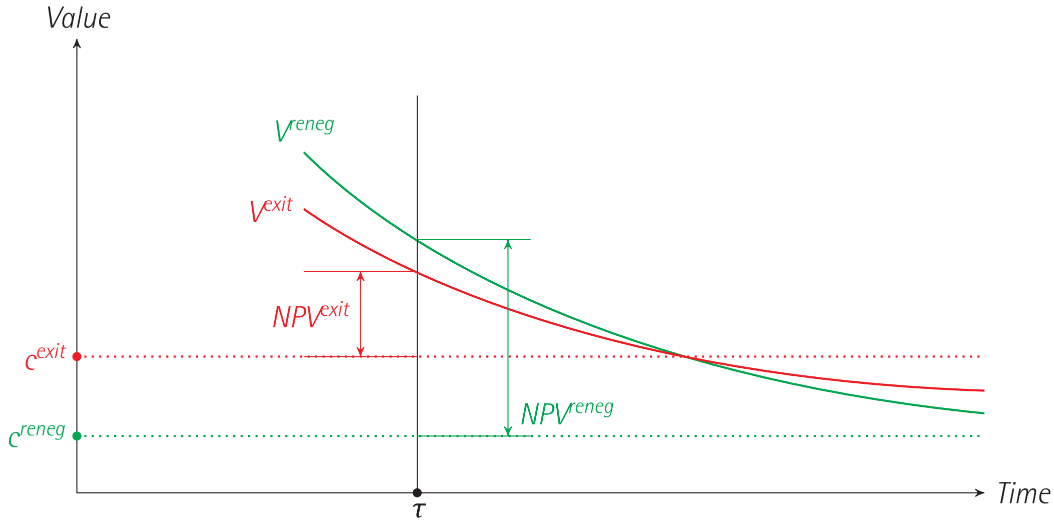
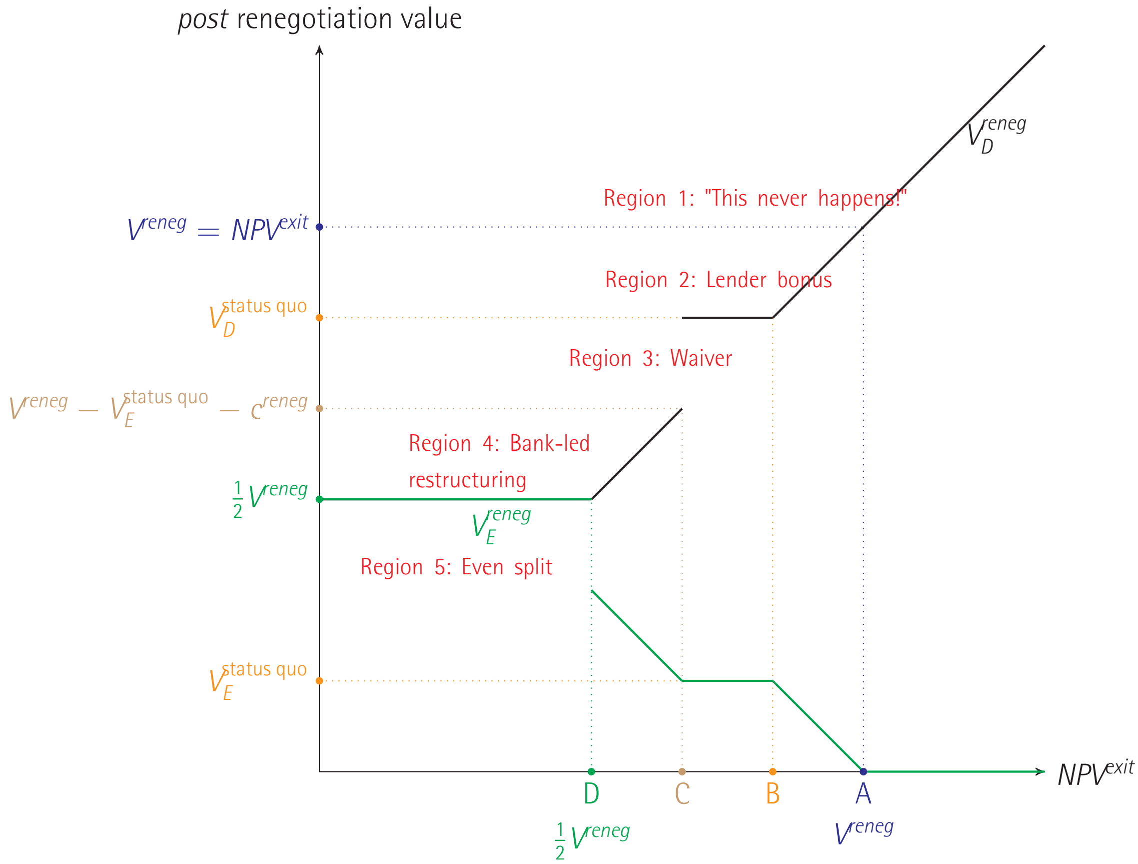


Figure 9: Outcome of renegotiation as a function of exit value following a hard default at the time τ: NPV^{exit} denotes the exit value at the time of default, and V^{reneg} represents the value of SPE in the renegotiation scenario; V_D^{reneg} and V_E^{reneg} represent the value of debt and equity in the renegotiation scenario, respectively; and $V_D^{statusquo}$ and $V_E^{statusquo}$ represent the value of debt and equity under the status quo debt schedule, respectively; c^{reneg} and c^{exit} represent renegotiation and exit costs, respectively; the black and green lines show the values of debt and equity upon renegotiation, respectively.



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(C_3) Debt holders never obtain less than equity holders.

If the first condition did not hold, at least one of the parties would have no incentive to participate in the renegotiation, and renegotiation would not occur. If the second condition did not hold, neither party would have an incentive to renegotiate, and renegotiation would not occur. The third condition simply postulates that debt holders, being effectively in control of the SPE upon a hard default, should be able to secure at least half of the value of the SPE in the renegotiation.

We denote the value of the SPE in the exit scenario as the *exit value* or NPV^{exit}, and its value in the renegotiation scenario as the *renegotiation value* or NPV^{reneg}.

From the point of view of the lenders, their *exit value* is the net present value of the CFADS in the exit scenario, net of any exit costs associated with taking over the SPE, finding a new equity investors or selling the loan, etc. By definition, the original equity owners loose their investment in the exit scenario i.e. their exit value is zero. Thus, in the exit scenario, the lenders' *exit value* is the same thing than the firm's *exit value*, whereas in the renegotiation scenario, both parties receive a positive share of the firm's *renegotiation value*.

In fact the *exit values* of debt and equity owners are also the lower bound of their *renegotiation values*, which provides an intuitive reason why renegotiation can happen.

As shown on figure 8, if the *exit value* of the SPE (V^{exit} = present value of the cash flows under lender control, net of exit costs) is less than its *renegotiation value* (V^{reneg} = present value of the cash flows under original ownership, when no renegotiation costs are incurred), then both debt and equity holders should be better off renegotiating the contract.

The two curves denoted by V^{exit} and V^{reneg} on figure 8 represent the value of the SPE in time, in the exit and renegotiation scenarios, respectively. These values are decreasing in time as the SPE approaches the end of its life (not shown on the figure). The exit and renegotiation *values* are equal to the firm's value in each scenario, minus exit and renegotiation *costs*, c^{exit} and c^{reneg}, respectively.

At time τ, a hard default has occurred and the parties must decide between the exit or renegotiation scenarii. On the figure, the exit value of the SPE at time τ (which is equal to the exit value of the lenders) is NPV^{exit}, while the renegotiation value is $NPV^{reneg} > NPV^{exit}$. In this case, both parties should prefer the renegotiation scenario. This is because not only the value of the firm is lower in the exit scenario but the exit costs are much higher than the renegotiation cost.

In other words, lenders and equity owner opt for the renegotiation or exit scenarii as a function of their relative renegotiation and exit values. If the renegotiation value of the SPE is sufficiently high compared to its exit value, debt holders can obtain more

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than their exit value by renegotiating, even after sharing a fraction of the SPE with the original equity owners, who would get nothing in the exit scenario.[23]

Thus, both the feasibility of renegotiation and its outcome are influenced primarily by the lenders' relative exit and renegotiation values of the SPE. In the extreme case, where the SPE is worth nothing in the exit scenario — because exit costs are very high for instance — debt holders have no choice but to renegotiate with existing equity holders.

Next, having discussed whether or not renegotiation or exit occur, we consider the different possible outcomes of the renegotiation scenario, which is also the most likely in practice.

Outcome of renegotiation post-hard default

Figure 9 graphically illustrates the outcome of renegotiation or post-renegotiation value on the vertical axis, as a function of the exit value NPV^{exit} on the horizontal axis i.e. a point further to the right denotes a higher exit value and for a given exit value, the different parties receive the value indicated on the vertical axis.

The black and green lines show the path followed by the values of debt and equity, respectively. Thus, as we move towards the right on the horizontal axis and their exit value increases, lenders are more likely to force an exit and leave the original equity investors with nothing, which is why the green line tends towards zero. For the same reason, the black line increases with the exit value, if its exit costs decrease.

Conversely, when the exit value is low because exit costs are high relative to the value of the SPE, the value of renegotiating increases and both parties find a mutually advantageous arrangement. Thus, moving towards the left of the horizontal axis, the green line increases and the black line decreases, as lenders agree to share some of the remaining value of the firm back with the original equity holders. As prescribed above however, lenders never share more than 50% of the firm's remaining value (point D on figure 9), since, at the time of the renegotiation, they have the effective control of the SPE.[24]

Next, different possible exit values create different equilibria splitting the value of the SPE post-default between debt and equity holders, found in five different "regions" of figure 9. Note that while at time t exit values are determined by exit costs, as time passes they are mostly determined by the project lifecycle and remaining value in the SPE. Thus, lenders may find themselves in different regions of figure 9 depending on when a hard default occurs.

We first highlight what we call the *status quo* value of debt and equity, $V^{status\ quo}_D$ and $V^{status\ quo}_E$ (points B) is the value of debt and equity if nothing is done, i.e. the value of debt and equity for the existing debt schedule without any restructuring or exit.

If exit the lenders' exit value is higher than (to the right of) point B (for example

23 - We assume that equity owners' opportunity cost of owning the project is zero and thus that they would always prefer a renegotiated outcome leaving them with a non-zero renegotiation value. In reality, equity holders would have to commit their time and exert effort running the firm. Hence, their exit value may not be zero but their opportunity cost running an alternative comparable project. Incorporating this non-zero opportunity cost is be one of the possible extensions of this model.

24 - This even split assumption could be relaxed, but it is reasonable for rational agents.

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because there is a very active secondary market with many buyers, so exit costs are low), they can extract more value from the renegotiation and leave a smaller share of the firm's value to equity holders than what they owned before default occurred. Two such cases can be distinguished corresponding to regions 2 and 1 on figure 9.

In the area called "Region 2", the lender's exit value lies between the status quo value of debt and the total value of the SPE i.e. there is scope for a mutually advantageous restructuring between lenders and equity owners: as a consequence, the renegotiated debt schedule is increased to at least the lenders' exit value, so that lenders will not exit and equity owners can retain the rest of the firm's value. In this case, a hard default leads to a net wealth transfer from equity to debt holders.

"Region 1" in the figure corresponds to a case where the lenders' exit value exceeds the original value of the firm.[25] Here, debt holders simply take over the firm, and equity holders get nothing. However, it can seem very unlikely that a hard default would ever occur in this case. One may also point to significant reputation risk for lenders actually choosing this course of action. In the implementation of the model described in chapter 4, this case is effectively ignored.

25 - Maybe the SPE is a natural resource project for which exploitable resources were vastly underestimated.

Next, if the exit value is lower than point B because exit costs are not negligible, the bargaining power of equity holders starts to increase, despite their initial share pledge giving lenders' the effective control of the SPE upon a hard default. Here, lenders cannot contemplate increasing the value of the original debt, but instead may have to take a "haircut" i.e. a loss. However, this loss is minimised by debt restructuring, especially if the loan has a long tail.

The probability of having to take a haircut (reported in chapter 5) depends on the relative bargaining power of lender and equity holders. Since the exit value is now low enough to create a guaranteed loss for lenders in the exit scenario, the question for them is to achieve a lesser loss through renegotiation.

First, in "Region 3", neither party has an incentive to initiate a restructuring and the outcome of renegotiation is still the *status quo* values, $V_E^{\text{status quo}}$ and $V_D^{\text{status quo}}$. This is because restructuring costs prevent a restructuring worth more than the *status quo* from taking place. Debt holders cannot gain from a restructuring and the outcome of renegotiation is to *waive* the Event of Default and to leave the original debt schedule unchanged. Similarly, the value of equity after incurring renegotiation costs is also less than its *status quo* value. Hence, both parties would prefer to write down the loss and continue with the pre-default debt schedule.

Moving further down the horizontal axis towards lower exit values (left of point C), in "Region 4" the exit value is small enough (relative to renegotiation costs) for debt holders to have an incentive to engage in a renegotiation and consider taking a "haircut" or loss. However, lenders may still achieve 100% recovery in this

3. Approaching the Valuation of Infrastructure Debt

case, depending on the characteristics of project, the point at which the renegotiation happens in the lifecycle and the size of the loan's tail. Moreover, their share of the total value of the firm is greater than that handed back to the original equity owners.

Finally, in "Region 5", the exit value is less than half of the original value of the SPE, and lender's bargaining power has further decreased vis-à-vis equity holders. Here, debt and equity owners renegotiate to share the value of the SPE equally. Note that in this case, if debt holders accepted that the value of the renegotiated debt be just above the exit value, they would get less than equity holders, which would violate the third renegotiation condition. The point here is that when there is no incentive to exit, lender's effective control of the SPE allows them to capture at least 50% of the remaining value. As before, high recovery rates are still possible depending on the total future value of the firm at the point.

A more technical description of the renegotiation model is presented in section 7.2.2 of chapter 7. In chapter 4, we further discuss the implementation of the debt renegotiation model and how the new debt schedule is computed upon an event of default, technical or hard.

Next, having projected the CFADS and the debt schedule in all states of the world, we discuss the valuation of the project's debt.

3.2.4 Valuation

To value the project's debt we use a structural model of credit risk. This first requires computing the distance to default of the SPE, which we show can be expressed as function of $DSCR_t$.

Distance to Default

In the Merton model, the firm's asset value follows a lognormal process with expected growth rate μ and asset volatility σ. In this case, the firm's distance to default can be approximated as (Crosbie and Bohn, 2003):

$$\text{Distance to Default} = \frac{[\text{MV}] - [\text{DT}]}{[\text{MV}].[\text{Vol}]} \quad (3.2)$$

where MV is the market value of assets, DT is the default threshold, and Vol is the **standard deviation of the annual percentage change in the asset value**.

The KMV model (Crosbie and Bohn, 2003) premises that DD is a sufficient statistic to arrive at a rank ordering of default risk, where the numerator in (3.2) expresses the firm's financial leverage or *financial risk*, while the denominator reflects its *business risk*. In other words, KMV assumes that differences between the credit risk of different companies are reflected in the value and volatility of their assets, as well as their capital structure, which are all present in the DD measure.

Following the definition of default in project finance given in (2.2), Distance to Default for infrastructure project finance loans at time t can be defined as:

$$DD_t = \frac{\text{CFADS}_t - \text{DS}_t}{\sigma_{\text{CFADS}_t}\text{CFADS}_t} \quad (3.3)$$

with $CFADS_t$, the cash flow available for debt service, and DS_t, the debt service at time t.

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Using the definition of $DSCR_t$ in equation 2.1, the above expression can be written as a sole function of the DSCR (see section 7.1.1 in chapter 7). Thus, we have:

$$DD_t = \frac{1}{\sigma_{DSCR_t}} \frac{\mathrm{DS}^{BC}_{t-1}}{\mathrm{DS}^{BC}_t} (1 - \frac{1}{DSCR_t}) \quad (3.4)$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the $DSCR$ value.

Hence, **the distribution of $DSCR_t$ together with the debt repayment profile** (the growth rate of the debt service defined by DS_{t-1}/DS_t) **are sufficient inputs to estimate the Distance to Default of project finance loans.**

Risk-Neutral Distance to Default

The DD computed above is the so-called "physical" or observed DD, and is independent of investors' attitudes towards risk.

In order to incorporate the effect of investors' risk preferences on credit risk measures and loan valuation, we must take into account the subjective probabilities that investors assign to future risky cash flows. As discussed in section 3.1.1, such probabilities are known as the risk-neutral probabilities. Under this measure, investors behaves as a risk neutral investor would and discounts the modified expected cash flows at the risk free rate.

In structural models, this risk neutralisation is done by mapping the physical distance to default to the risk-neutral distance to default, using a probability transform derived from the Merton model. This probability transform decreases the physical distance to default by investors' required risk premium for one unit of risk, to obtain the risk neutral distance to default. That is,

$$DD^*_t = \frac{\mathrm{CFADS}_t - \mathrm{DS}^{BC}_t}{\sigma_{\mathrm{CFADS}_t} \mathrm{CFADS}_t} - \frac{\mathrm{Premium}}{\sigma_{\mathrm{CFADS}_t}} \quad (3.5)$$

where DD^*_t is the risk neutral distance to default.

As discussed above, in incomplete markets, the no-arbitrage principle does not lead to a single pricing measure but instead to weak bounds on value, which can be strengthened using approximate arbitrage models.

Approximate arbitrage bounds are obtained by arbitrarily limiting the attractiveness of investment opportunities (measured by the required Sharpe ratio in our model i.e. the risk/reward trade-off demanded by an individual investor) to rule out investments that are either too risky or too attractive to be expected to survive at price in the market.

Assuming risk-averse investors, the lower bound for the Sharpe ratio is zero, and the upper bound can be set to a multiple of the Sharpe ratio of a broad market index, or derived from a model of agents' preferences under equilibrium. Within these arbitrage bounds, different investors may demand different prices for the same security.

Hence, the mapping between physical and risk neutral distributions is not unique in incomplete markets. The range of risk neutral distributions consistent with the no-arbitrage principle depends on the extent

3. Approaching the Valuation of Infrastructure Debt

to which the risks of individual instruments are spanned by securities traded in a liquid market. As the proportion of these unhedgable risk decreases, the range of the pricing bounds also shrinks, and in the limit, when all risk is hedgable using traded instruments, the no-arbitrage rule implies that the range of risk neutral distributions must converge to a unique probability measure.

Finally, once the cash flows to debt holders in all states of the world have been determined and can be discounted under a range of risk-neutral measures, the total value of the debt can be computed using the Black-Cox decomposition.

Black-Cox Decomposition

The Black Cox decomposition (Black and Cox, 1976) was devised to value corporate securities when firms can be restructured, which typically occurs during financial distress (to avoid bankruptcy), or when they are sufficiently profitable (to benefit from high credit quality).

Under the Black Cox decomposition, it is assumed that a firm is re-structured if its value passes a lower boundary (i.e. financial distress) or if an upper limit (i.e. high level of free cash flow).

The value of a firm's debt is then derived from four sources: 1) the payout at maturity, 2) the payout if the firm is restructured at the lower boundary, 3) the payout if the firm is restructured at the upper boundary, and 4) its payout before any of the above events. These four sources of value are shown in the figure 3.2.4.

The Black Cox decomposition is general enough to value any corporate security, and provides an intuitive way to incorporate the effects of firm reorganisations. Hence, it is a natural choice for valuing securities issued by project companies that are frequently restructured.

However, while the original Black-Cox decomposition assumes that restructurings happen when the total value of the firm reaches a lower or an upper boundary, PF SPEs reorganisations are determined not by the total value of the SPE at a point in time but by the CFADS at each point in time.

Therefore, we modify the Black-Cox decomposition to take into account this difference. In our model the Black Cox decomposition is used to decompose the value of PF debt into its four components:

1. Its value at the maturity date, if the debt has not been reorganised before then.
2. Its value if the debt is restructured at some lower boundary. This can occur if the CFADS falls below the default threshold;
3. Its value if the debt is restructured at the upper boundary. This can occur if the CFADS exceeds expectations and the project company can refinance.
4. The value of the debt payouts prior to any of the three events described above.

3.2.5 Putting It All Together: Total Value of Debt

In the Black Cox decomposition, the task of valuing a security largely reduces to identifying the four payout functions of the

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security, and then determining the present value of those payouts.

It should be stressed that in case of project finance loans, all payout functions are not determined by the original debt contract. In particular, the payout at the lower boundary (default threshold) is not specified in the original contract, but is determined through debt renegotiation.

Nevertheless, once the payout functions have been determined, we can discount the security's payouts to determine its present value. The appropriate discount rate in the case of risk neutral valuation is the risk free rate, as the effects of risk preferences have already been incorporated in the risk neutral probability measure.

That is, one can simply compute the expected payouts of security at every point in its life under the risk neutral measure, and then discount them at the risk free rate to determine its fair value. For a mathematical illustration, see section 7.2.1 in chapter 7.

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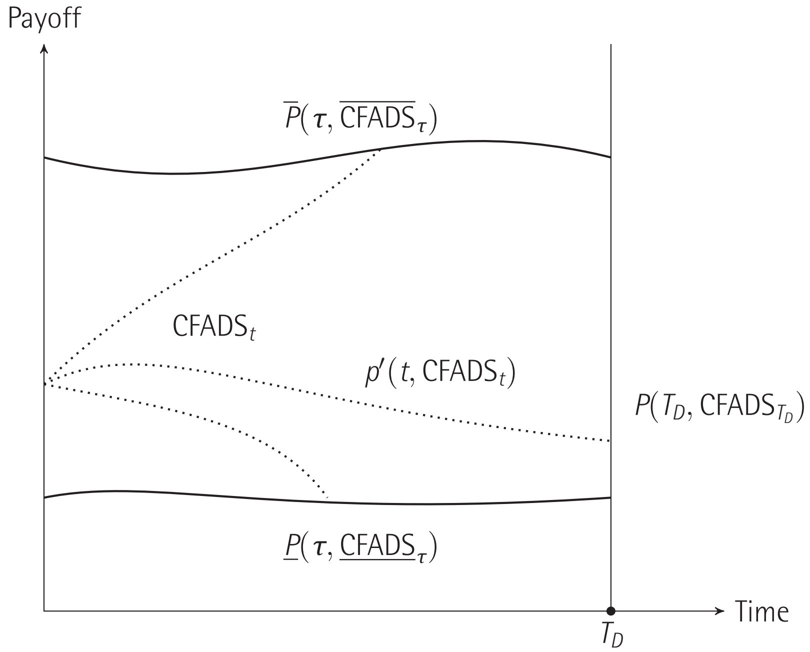
Figure 10: Black-Cox decomposition at one point in time. $\overline{P}(\tau, \overline{CFADS_\tau})$ is the payout function if CFADS hits the upper boundary, $\underline{P}(\tau, \underline{CFADS_\tau})$ is the payout function if CFADS hits the lower boundary, $P(T_D, CFADS_{T_D})$ is the payout function at the maturity of the debt, and $p'(t, CFADS_t)$ is the payout function before CFADS hits any of the boundaries or reaches maturity. In the presence of renegotiations, one would need to perform a new Black-Cox decomposition every time CFADS hits the lower boundary with updated payout functions, $p'(t, CFADS_t)$, $P(T_D, CFADS_{T_D})$, $\overline{P}(\tau, \overline{CFADS_\tau})$, $\underline{P}(\tau, \underline{CFADS_\tau})$ determined through renegotiation.



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In this section, we implement our model for two generic types of infrastructure projects.

4.1 Cash Flow Model

To model the DSCR profile, we distinguish between the two generic families of infrastructure project financing structures which we call *Merchant infrastructure* and *Contracted infrastructure*.

We focus on revenue risk as a heuristic to distinguish between relatively homogenous families of cash flow dynamics, because project finance SPEs typically manage their construction and operating costs through fixed-price contracts, which transfer most risks to subcontractors. As a consequence, their revenue risk profiles is the significant explanatory variable that caotures different levels of credit risk. For example, Blanc-Brude and Strange (2007) and Blanc-Brude and Ismail (2013) find that revenue risk is the most important statistical determinant of credit spreads in infrastructure project loans, controlling for loan characteristics (maturity, size, etc) and the credit cycle.

As discussed in section 3, merchant infrastructure projects refer to projects that are exposed to market risk, for example a toll road. These projects are generally structured at financial close to have a rising DSCR mean — implying an increasing DSCR volatility — and a longer loan tail.[26] The rising DSCR profile implies that creditors are paid back faster than equity owners. This is to minimise any adverse effects of increasing CFADS volatility on loan repayment. Project revenues in the loan's tail act as security for lenders, and allows rescheduling of debt to recover any losses incurred due to financial difficulties during the original life of the loan. In short, creditors demand faster repayments and higher collateral to protect themselves against higher CFADS volatility — created by higher revenue risk — in merchant infrastructure projects.

Contracted infrastructure refers to lower risk projects that receive pre-agreed payments from a client (e.g. the government or a utility) for a fixed period of time, and have very little, if any, exposure to market risks. With lower CFADS volatility, these projects are typically structured to have a constant or flat DSCR in time, and a shorter tail, as lenders need less protection against default. Examples of these projects include schools, hospitals and other types of social infrastructure projects, as well as projects financed on the back of a take-or-pay purchase agreement, by which a client commits to buying (or pay for) the project's output, at a price that can be set in advance as well. In short, creditors are willing to accept longer repayment periods and lower collateral when lending to contracted infrastructure projects due to their lower credit risk. The main revenue risk in contracted infrastructure is counterparty risk i.e. the risk that the buyer defaults or reneges on its obligations.

We note that a number of infrastructure projects belong to a hybrid category which mixes both merchant and contracted revenue risk e.g. so-called shadow toll road projects or energy projects that commit part

26 - The loan's tail is the period between the debt and project maturities is referred to as the tail of the loan.

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of their production capacity in a purchase agreement and sell extra capacity in the spot market. We do not explicitly address such structures in this paper, but we note that these structures could be modelled as a portfolio or combination of contracted and merchant projects.

Table 1 provides our characterisation of our the generic project structures. Both projects last for 25 year. The merchant project has a 5 year construction period, is financed with 75% leverage,[27] the loan is repaid between year 6 and 19, hence a tail of 6 years.

27 - We define leverage as the ratio of the market value of the loan to the market value of the SPV at financial close. Hence, the leverage is sensitive to the risk preferences of the investor. Different choices of risk preferences (Sharpe ratio) may lead to different values of debt and SPV, and hence the leverage may change. The leverage given in the table is for a benchmark investor with a Sharpe ratio of 1.

The contracted infrastructure project has a 3 year construction period, is financed with 90% leverage, and repays the loan between years 4 and 23, leaving a tail of 2 years. Total initial debt is normalised to $1,000.

At this stage, before empirical observations can be made, we model the DSCR for the merchant project using a lognormal distribution with a constant mean return (increase) of 1%, a constant volatility of returns of 3%, an initial DSCR of 1.4, and 20% volatility of the initial DSCR. That is, *ex ante*, the DSCR of the project is expected to be 1.4 with a standard deviation of 20% immediately after construction, and is then expected to rise lognormally with 1% mean return and 3% volatility in returns.

Mathematically, the distribution of this **rising DSCR family** is written:

$$\frac{d(DSCR_t)}{DSCR_t} = \mu dt + \sigma dW_t, \tag{4.1}$$

The DSCR for the contracted project is modelled using a normal distribution with a mean DSCR of 1.2, and a volatility of 8%. Hence, *ex ante*, the DSCR for the contracted project is expected to be normally distributed around 1.2 with a standard deviation of 8% for the entire life of the loan.

The distribution for this **flat DSCR family** is given by:

$$DSCR_t = E[DSCR] + \sigma(DSCR) dW_t. \tag{4.2}$$

We list the model parameters for the two DSCR distributions in table 2. Figure 11 shows the projected DSCRs for both families of DSCR dynamics.

The base case DSCR is available only until the original maturity of the loan. However, in order to take into account the value of tail, one needs to project CFADS in the tail. For this purpose, we assume that the CFADS distribution does not change after the original loan maturity.

In the two examples discussed above, the CFADS follows the same distribution as the DSCR, as the debt payments are constant in time. Thus, we project the CFADS in the tail using the same distribution that was used during the life of the loan. This is a simplifying assumption, as project finance debt service is often 'sculpted' but this would be a sufficiently close approximation across a basket of loans.

In the rest of this chapter, we describe the technical implementation of the valuation model that we described in more intuitive

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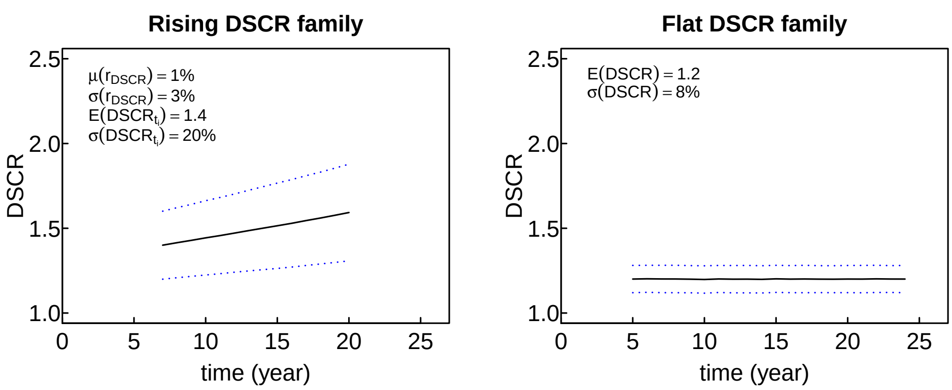
Table 1: Merchant and Contracted Infrastructure characteristics

Project type	Construction period	Tail length	DSCR profile	Project maturity	First payment	Final payment	Base case debt IRR
Merchant	5 year	6 year	Rising	25	Year 6	Year 19	4%
Contracted	3 year	2 year	Flat	25	Year 4	Year 23	3.5%

Table 2: DSCR models for the two DSCR families.

DSCR profile	DSCR distribution	Mean Return	Volatility of returns	Initial expected DSCR	Volatility of initial DSCR
Rising	Lognormal	1%	3%	1.4	20%
Flat	Normal	NA	NA	1.2	8%

Figure 11: DSCR models for Merchant and Contracted infrastructure



terms in section 3. Non-technical readers may skip this part and proceed to the results in chapter 5.

4.2 Risk Neutral Measure

4.2.1 Distance to Default and DSCR

The distance to default is a standard metric used both in practice and in structural models to measure credit risk. Distance to default (DD) has a unique relation to the probability of default (PD), given by

$$\mathrm{PD}_t = \Phi(-\mathrm{DD}_t) \tag{4.3}$$

where $\Phi(.)$ is the standard normal cumulative density function (CDF).

As we argued previously, in project finance, distance to default can be computed with the knowledge of the distribution of $DSCR_t$ as

$$DD_t = \frac{1}{\sigma_{DSCR_t}} \frac{\mathrm{DS}^{BC}_{t-1}}{\mathrm{DS}^{BC}_t}\left(1 - \frac{1}{DSCR_t}\right), \tag{4.4}$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the $DSCR$ value. The derivation of this equation is shown in section 7.1.1 in chapter 7.

Thus, by documenting the distribution of $DSCR_t$ we can compute the DD_t metric, which is instrumental in structural models of credit risk.

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4.2.2 Mapping Between the Physical and Risk Neutral Distributions of $DSCR_t$

As argued before, in the presence of market incompleteness and frictions, asset prices would not necessarily converge to a unique price, and different investors may pay different prices for the same asset. This variability in prices stems from the heterogeneity of investors' risk preferences. We incorporate this heterogeneity through the subjective probabilities that investors assign to future risky cash flows, the so called risk-adjusted or risk-neutral probability measure.

To arrive at the risk neutral distribution, we start with the mapping between risk neutral and physical distance to default, which is given by Kealhofer (2003) and Duffie and Singleton (2003):

$$DD^*_t = DD_t - \frac{\mu - r}{\sigma} \tag{4.5}$$

And the risk neutral distribution of the DSCR for the rising DSCR family is given by

$$DSCR_t = DSCR_{t-1} e^{(\mu - \lambda\sigma - 0.5\sigma^2) + \sigma dW_t} \tag{4.6}$$

where μ and σ are the mean and volatility of the physical distribution of $DSCR_t$ for the rising family, and λ is the Sharpe ratio or risk/return trade-off required by an investor as a function of its risk, liquidity and other preferences and existing allocation to infrastructure debt. This derivation is described in section 7.1.2 in chapter 7.

Likewise, the risk neutral distribution of the DSCR for the flat DSCR family is given by

$$DSCR_t = (E[DSCR_t] - \lambda\sigma) + \sigma dW_t \tag{4.7}$$

where $E[DSCR]$ and σ are the mean and standard deviation of the physical (real world) distribution of $DSCR_t$ for the flat family, and λ is defined as above.

4.2.3 Choice of Bounds on Required Risk Premium

Before proceeding with the valuation model, we discuss our choice of bounds on the required Sharpe ratio λ. We argue that the investors' Sharpe ratios would lie in a band between 0 and 2.

The lower bound of 0 corresponds to an investor that requires no premium above the risk-free rate for investing in the PF loan. This could be the case for an investor that holds a well-diversified portfolio so that the marginal contribution of the loan to an existing portfolio is negligible e.g. the State. The upper bound corresponds to an investor that demands a 200 basis point premium above the risk-free rate *for every unit of risk* in the PF loan. In this setting one unit of risk corresponds to one standard deviation of $DSCR_t$.

Next, we elaborate why the required Sharpe ratios are unlikely to exceed these bounds.

Annualised Sharpe ratios for market indices typically fall below 1. Conversely, the largest Sharpe ratios are often exhibited by hedge funds. Even for high performing hedge funds, the only instances where the Sharpe ratio may exceed 2.0 are when their returns are not normally distributed (Kat and Brooks, 2001). Non-normal distributions exhibit higher moment risks, such as negative skewness, high kurtosis, and the

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Sharpe ratio (which only takes into account the first two moments) can underestimate the riskiness of such investments.[28]

Since we assume normal distribution for returns in our example,[29] we argue that if PF loans offered Sharpe ratios above this upper limit of 2, they would become too attractive, and that such loans would soon disappear from the market. Therefore, in equilibrium, the Sharpe ratios for PF loans would lie between 0 and 2.

The bounds on Sharpe ratios (or other risk reward ratios) can also be obtained from models of agents' preferences that derive the corresponding bounds on risk/reward ratios. For example, the arbitrage model assumes monotonicity of preferences[30] and the corresponding bounds on the Sharpe ratio turn out to be $(0, \infty)$.

However, the assumption of monotonicity of preferences is a weak assumption, and the resulting bounds are very wide. Approximate arbitrage models make more stringent assumptions on investors' risk preferences, and consequently the bounds can be shrunk.[31] Cochrane and Saa-Requejo (2000) shows that even with high levels of risk aversion and volatility in future levels of consumption, Sharpe ratios do not exceed 1.72. Hence, our choice of an upper limit of 2.0 seems justified from both an applied and a theoretical perspective.

4.2.4 Bounds on $DSCR_t$ and DD_t

Using the bounds discussed above, shows the risk-neutral distribution of the two DSCR families on figure 13 and the risk-neutral distance to default for a range of levels of risk aversion in figure 12. As expected, distance to default for the lower bound on risk aversion (blue curve) always lies above the distance to default for the upper bound on risk aversion (red curve).

Moreover, distance to default changes with time for the rising DSCR profile, as its mean and standard deviation change in time, and stays constant for the flat DSCR profile as its mean and standard deviation are constant in time.

The level and slope of the distance to default curve is determined by investors' level of risk aversion (their required Sharpe ratio). As risk aversion increases, investors discount more to compensate for the increasing volatility of the DSCR, further decreasing the risk neutral distance to default and the slope of the distance to default curve. Distance to default for the upper limit on the Sharpe ratio is always negative, implying a more than 50% risk-neutral probability of default from the beginning, that approaches 100% after a few periods (See equation 4.5).

For a benchmark investor with a Sharpe ratio of 1, the risk-neutral distance to default starts at about 0.8 and ends at about -0.8, implying a risk neutral probability of default that starts at about 20% and reaches 80% at the maturity of the loan.

4.2.5 Decomposition of Risk into Traded and Non-Traded Components

We have argued earlier that incomplete markets imply that some risks can be hedged in markets while others cannot.

28 - For example, Long-Term Capital Management (LTCM) exhibited a Sharpe ratio of 4.35 before its demise in 1998 (Lux, 2002). However, as is now well known, this hedge fund was exposed to some extreme risks, and its return distribution was highly non normal.

29 - For non-normal distributions, the bounds can be specified using other risk reward ratios, such as the gain-loss ratio introduced by (Bernardo and Ledoit, 2000).

30 - Monotonicity of preferences implies that all investors would prefer more to less.

31 - For a detailed exposition of approximate arbitrage models, see Cochrane and Saa-Requejo (2000) and Bernardo and Ledoit (2000), which demonstrate the determination of bounds on Sharpe ratios, and gain/loss ratio, respectively.

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Figure 12: Risk neutral distance to default for the two DSCR families.

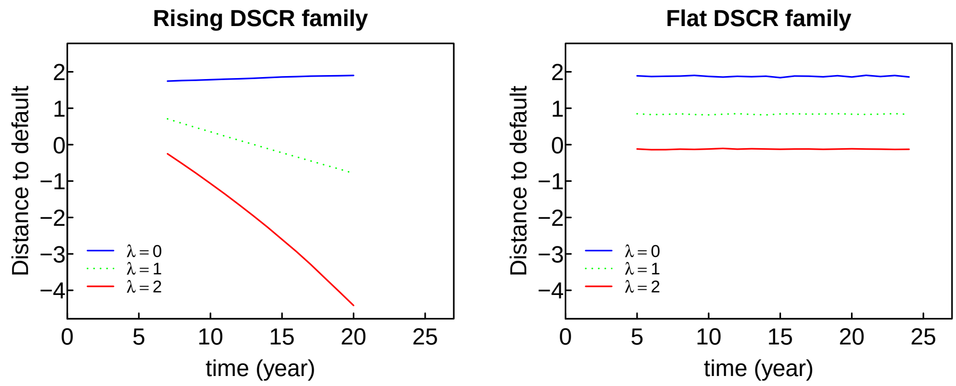
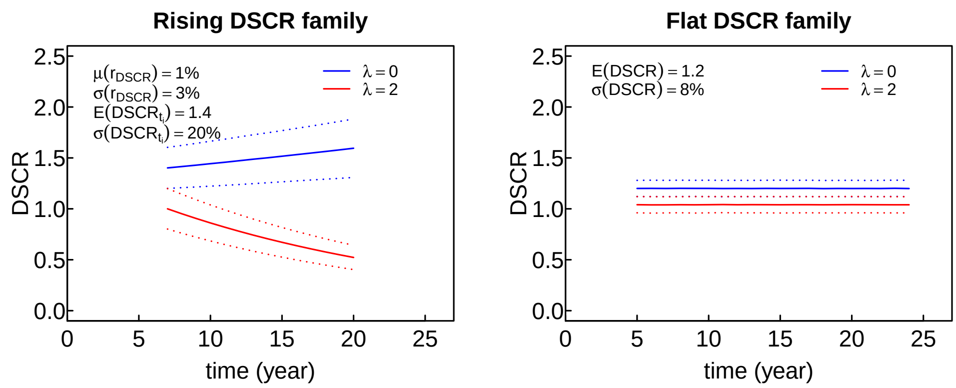


Figure 13: Risk neutralised DSCR distributions for the economic and social infrastructure projects.



In order to estimate the required prices of risk more precisely and tighten the pricing bounds, we could separate total risk into a traded (hedgable) and an untraded (unhedgable) components. This is equivalent to separating PF loans into two portfolios: a traded portfolio, and an untraded portfolio. This separation of risks serves two main purposes:

1. The required prices for hedgable risks can be set equal to the premium earned by the traded portfolio, irrespective of investors' preferences. Hence, the model prices would stay consistent with the market prices and would not lead to any arbitrage opportunities. This also takes into account the extent of market incompleteness. As the fraction of traded risk increases, the Sharpe ratio bounds would shrink to the Sharpe ratio of the traded portfolio, and in the limit when all risk is traded, the required Sharpe ratio would be unique.
2. The required prices for unhedgable risks can be calibrated to the observed PF debt prices, and the model can be used to learn about variations in risk preferences across investors and in time. Thus, the impact of different regulatory

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requirements, liability structure, and other factors that may affect investors' risk preferences can be modelled, and calibrated to observed prices.

After separating into traded and untraded components, the required Sharpe ratio can be written as (see section 7.1.3 in chapter 7):

$$\lambda = w_{t-1}^{T} \frac{\sigma^T}{\sigma} \lambda^T + w_{t-1}^{N} \frac{\sigma^N}{\sigma} \lambda^N, \qquad (4.8)$$

where $w_{t-1}^{T(N)}$ is the fraction of the period's CFADS that is replicated by the traded (non-traded) portfolio, $\sigma^{T(N)}$ is the standard deviation of traded (non-traded) portfolio, and $\lambda^{T(N)}$ is the Sharpe ratio of traded (non-traded) portfolio.

The determination of traded and non-traded fraction of the CFADS is an empirical task, and in our example we assume all risk is non-traded.

4.3 Debt Rescheduling Upon Default

In this section, we discuss how the debt schedule changes upon default in more details. We consider both a technical default and a hard default at time $t = 10$. As the same procedure is followed in updating the debt schedule, we only discuss the procedure for the rising DSCR family.

We assume that the realised CFADS at $t = 10$ turns out to be \$131, barely sufficient to satisfy the scheduled debt payment of \$130.7737. Hence, the SPV goes into a technical default. As highlighted before, debt holders have the right to reschedule their outstanding debt in this instance. Hence, according to our renegotiation model, debt holders reschedule the outstanding debt if the market value of the *rescheduled debt* (net of rescheduling costs) exceeds the market value of the existing debt schedule.

In our example, the amount of debt outstanding at $t = 10$ is $\$1,003.595$ (the present value of future debt payments discounted at the initial IRR of the loan). We assume that debt holders reschedule their debt under a constant amortisation profile. Note, however, that debt holders may reschedule the debt in numerous other ways and that different assumptions could be used.

Figure 14 shows the rescheduled debt upon technical default for two scenarios: 1) when rescheduling costs are \$100 (red curve), and 2) when rescheduling costs are \$10 (blue curve). In the first case, due to relatively high rescheduling costs, debt holders cannot find any debt schedule with a constant amortisation profile that exceeds the existing debt schedule in market value. Hence, the debt schedule remains unchanged. In the second case, when rescheduling costs are relatively low, debt holders find that the optimal debt schedule is the one with the longest maturity, i.e. the maturity of the project, and the corresponding fixed debt payment is \$93.4413. Hence, the debt holders reschedule their debt until project maturity.

We note two things: first, both debt schedules — the initial one with a constant

4. Model Implementation

Figure 14: Debt rescheduling upon technical default, BC=base case, C=rescheduling cost

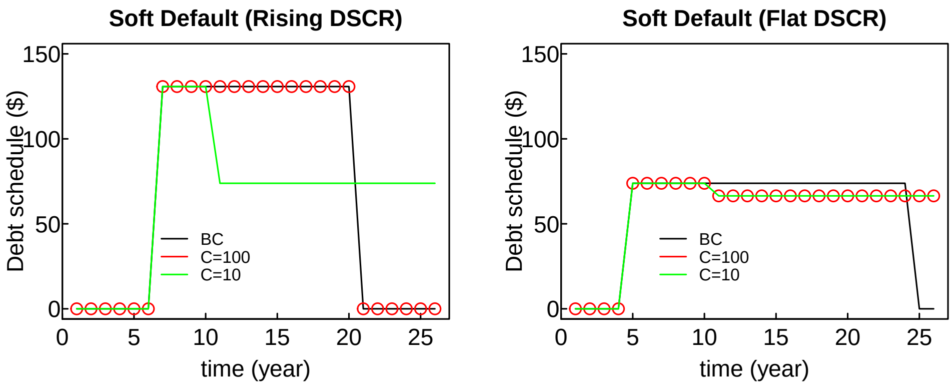
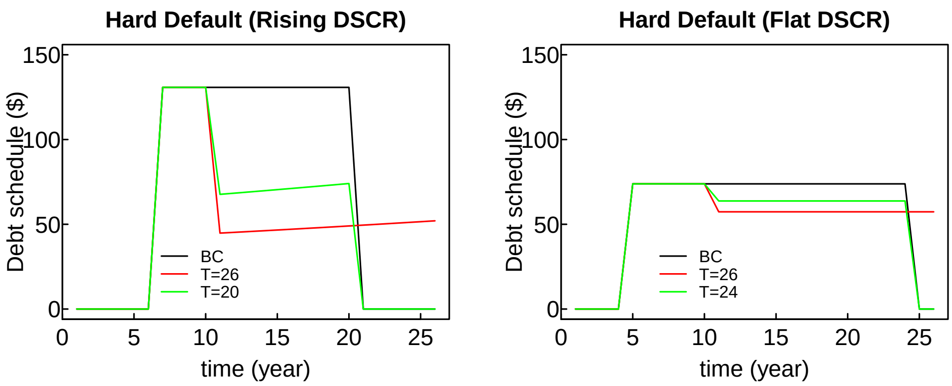


Figure 15: Debt rescheduling upon hard default, BC=base case, T=maturity of the new debt schedule



debt payment of $130.7737 until year 20, and the new one with a constant debt payment of $93.4413 until maturity – have the same amortised value of $1003.595 at $t = 10$. Yet, the two debt schedules have different *market values*. The reason is that their market value takes into account the probability of default, and the liquidation value, while the amortised value does not. Hence, by selecting a lower debt payment as compared to the previous debt schedule, debt holders decrease the SPEs' probability of default, and increase the market value of the debt, without affecting the amortised value of debt. **This is the value of the step-in option.**

Second, somewhat counter-intuitively, the optimal debt schedule is the one with the longest possible maturity. The reason, again, is that the market value of debt is affected by the SPE's liquidation value and the probability of default. If the liquidation value of SPE is small, the debt holders are better off minimising the probability of default even if that involves increasing the duration of the debt. Hence, the optimal debt schedule in this case happens to be the one with the longest debt maturity.

4. Model Implementation

Next, we consider the rescheduling of debt upon a hard default at $t = 10$. This time, we assume that the realised CFADS upon default is \$110. Figure 15 shows two ways to reschedule debt upon hard default for a liquidation value of \$601.1687. The red line shows the rescheduled debt when debt holders choose to reschedule debt until the original maturity of the debt (20 years), and the blue line shows the rescheduled debt when debt holders choose to reschedule debt until the maturity of the project (25 years). In this case, while the two debt schedules have different maturities, both have the same market value, as they are determined such that their market value equals the liquidation value of the SPE. Yet, the two debt schedules may not have the same amortised values, which may also be different from the amortised value of the existing debt schedule.

This is the main difference between the rescheduling upon a hard and a technical default. Upon a technical default, the amortised value of debt stays fixed, as debt holders are constrained by the existing contract to only reschedule the outstanding amount of debt. However, **the market value of debt upon a technical default may well exceed the market value of the original debt**.

Upon a hard default, the market value of debt is fixed, as it is determined by the outcome of renegotiation – as discussed in chapter 3 – and lenders cannot increase the value of their debt by changing the debt schedule.

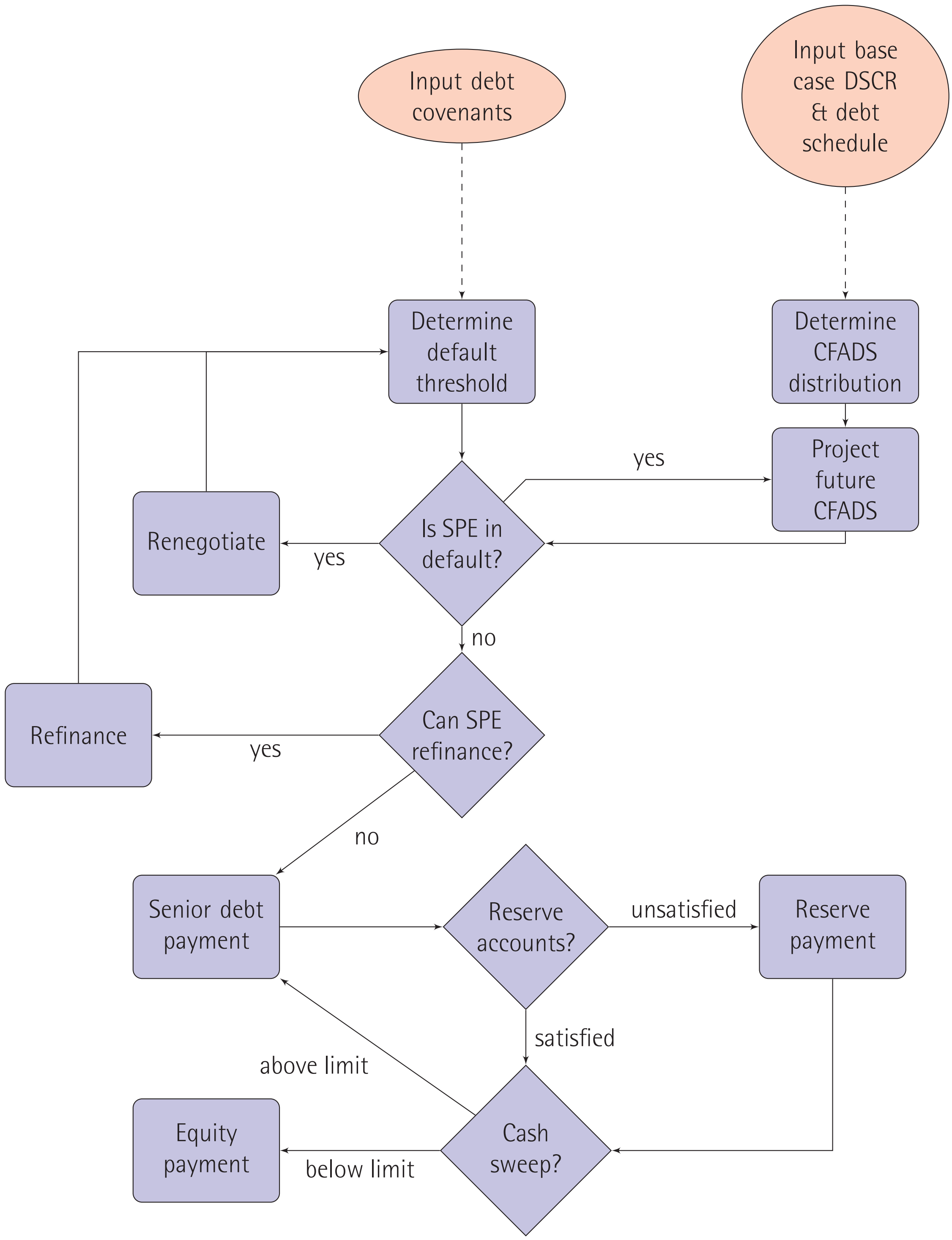
4.4 Algorithm

In this section, we describe an algorithm for the numerical implementation of our model, as illustrated by figure 16. The main steps in implementing the framework are

1. Obtain the base case debt schedule;
2. Obtain the base case DSCR profile;
3. Determine the CFADS distribution: Using the DSCR model and base case debt schedule, we can infer the CFADS distribution using equation 3.1;
4. Risk neutralise the distribution of the CFADS: Select a required Sharpe ratio, and shift the original DSCR (or CFADS) distribution accordingly;
5. Obtain debt covenants: Debt covenants may contain reserve accounts, cash sweeps and clawback provisions etc. and include the technical default threshold: the threshold below which lenders have the right to step in and reschedule the debt;
6. Project CFADS paths for future periods using the distribution obtained above.
7. Determine if the SPE is able to refinance: for each projected CFADS path, determine if the SPV has transitioned into a sufficiently low risk environment where it can refinance its debt;
   - If refinancing is possible, i.e. if the projected CFADS exceeds the refinancing threshold, determine the new debt covenants (debt service schedule, reserve account requirements etc.). All debt covenants need not change, and the only change may be in the debt service schedule and the default threshold;

4. Model Implementation

Figure 16: Flow chart of the determination of cash flows to debt holders



4. Model Implementation

8. Determine if the SPE is in default: Compare the projected CFADS for each period with the default threshold;
    - If the SPE is in technical default, debt is rescheduled if a new debt schedule can be found that exceeds the existing debt schedule's market value;
    - If the SPE is in hard default, the new debt schedule is determined based on the outcome of renegotiation model;

9. Construct the cash flow waterfall with existing debt covenants: making payments according to the priorities established in the debt covenants, which would include payments to debt holders, reserve accounts, and equity holders;
10. Once cash flows to the debt holders have been projected, the present value of these cash flows is calculated under the risk-neutral probability measure using risk-free discount rates.

The code for the two examples provided in the paper is implemented in R, and consists of five key functions:

1. A function to project CFADS under the risk neutral DSCR distribution;
2. A function to reschedule debt upon technical default;
3. A function to reschedule debt upon hard default;
4. A function to construct the cash flow waterfall for a given debt schedule when the SPE is not in default;
5. A function to call the above functions, performs a Monte Carlo simulation, and computes the risk and return measures.

The key inputs, calculations, outputs and parameters of the model are as follows:

Inputs

1. Base case debt service schedule;
2. Base case DSCR for each period;
3. Loan and project maturities;
4. Technical default triggers, including the minimum required DSCR;
5. Non-operating reserve account requirements as specified in the debt covenants. These may include debt service reserve account (DSRA), and special reserve accounts;
6. Cash sweep threshold amount i.e. the maximum amount that can be distributed to equity holders in a period;
7. Fraction of CFADS that can be replicated by traded securities;
8. Volatility and Sharpe ratio for the traded component of CFADS;
9. Term structure of risk free interest rates.

Calculations

1. Determination of risk neutral probability distribution of DSCR and CFADS;
2. Determination of new base case debt schedule upon technical and hard defaults;
3. Determination of future cash flows to debt holders for a given debt schedule;
4. Determination of performance measures.

Outputs

1. Marginal probabilities of default;
2. Per period expected losses, VaR and cVaR;
3. Effective duration of debt;

4. Model Implementation

4. Present value of expected losses and recovery rates;
5. Yield and z-spread.

Parameters

1. Parameters of the DSCR distribution;
2. Required Sharpe ratio;
3. Liquidation, renegotiation and rescheduling costs;
4. Refinancing threshold.

5. Results

5. Results

In this section, we compute risk and return measures for our generic merchant and contracted infrastructure project debt. We also report the outcome of the debt renegotiation model.

5.1 Model Assumptions

- Equity dividends are locked up if DSCR falls below 1.10, and technical default is triggered if DSCR falls below 1.05;
- Liquidation costs are 60% of initial value of debt;
- Renegotiation costs are one half of liquidation costs;
- Restructuring costs are one third of liquidation costs;
- We ignore the Region 3 in figure 9, in which no renegotiation could take place, and assume that debt value is equal to the liquidation in this region (this is to simplify numerical computation and in unlikely to have any significant influence on the risk profile).
- We ignore any termination payment due by a "grantor" — typically the public authority that granted the concession to the SPE — which would automatically create a floor of lender's exit value. Such guarantees exist in a number of projects but are not systematically found in project finance. However, they could easily be implemented as an extension of the model.

5.2 Risk Measures

In this section, we compare the risk profiles of the two DSCR families. This includes a comparison of debt payments, probability of default, per period losses and value-at-risk, and per period conditional value-at-risk (expected shortfall).

Figure 17 compares the CFADS, and mean debt payments for the two families. We see that on average, in both cases, mean debt payments gradually fall below the base case debt schedule, but exceed the base case debt schedule in the tail, hence reducing losses incurred in the earlier periods.

5.2.1 Risk Profile Dynamics

We find relatively low levels of credit risk (compared to the current treatment of infrastructure debt in Solvency-II for instance) and we note that the dynamics implied by our model and parameter values are comparable with the the level of risk and the type of credit risk transition reported by ratings agencies (see for instance Moody's, 2014).

The probability of (technical or hard) default (PD) decreases rapidly for the rising DSCR family since the DSCR has constant positive mean, which makes default unlikely in the later periods of the project's life. PD is mostly constant for the flat DSCR family, because the DSCR stays at the same level and hence the likelihood of default does not decrease over time. Thus, as we argued earlier, a correctly calibrated distribution of $DSCR_t$ is a good predictor of PDs, especially if not enough data is available to use reduced form models.

We note that the difference between the probability of all defaults default (black line) and the probability of hard defaults (under

5. Results

Figure 17: Comparison of mean debt payments and CFADS.

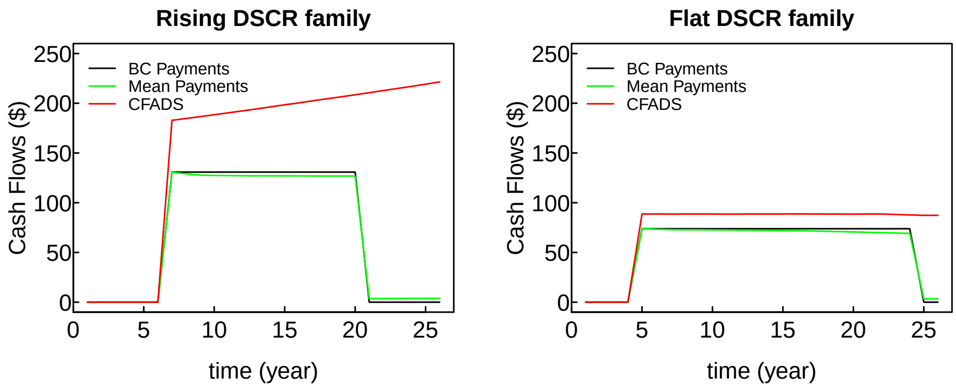


Figure 18: Comparison of probability of technical or hard default (PD), hard default only (Moody's definition) and probability of death (no recovery), for the two DSCR families.

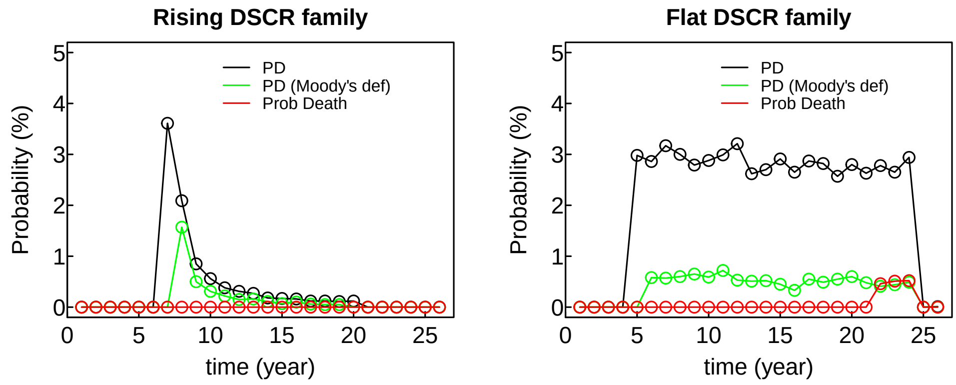
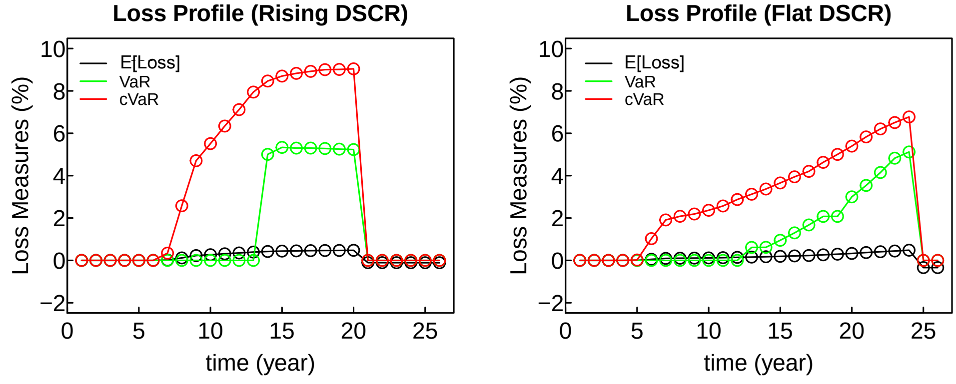


Figure 19: Comparison of expected loss, VaR, and cVaR for the two DSCR families.



5. Results

Moody's definition [32]) (green line) is significant in the case of flat DSCR family. This also makes the step-in option – the right to step in and reschedule debt upon a technical default – more valuable in the case of flat DSCR family than that of the rising DSCR family.

Figure 19 compares the loss profile (expected loss, VaR, cVaR) for the two DSCR families. In the case of the flat DSCR family, expected loss (EL), VaR and cVaR all rise towards the maturity of the debt. In the case of rising DSCR family, while mean EL do rise, VaR and cVaR stay constant near the maturity of the debt.

The rising trend in EL can be explained by the increasing cumulative probability of default. As more hard defaults occur over time, debt holders get a hair cut, and post-default mean debt payments decrease. Therefore, mean debt payments near the maturity of the loan reflect the accumulated effect of hair cuts due to all the hard defaults in the previous periods. This is why mean debt payments are lower near debt maturity (as seen in figure 17), and mean losses are higher, even for the rising DSCR family, for which the marginal default probability near maturity is close to zero.

The difference in the VaR and cVaR trend stems from the different tail values of the two families. In the case of flat DSCR family, the lower tail value and relatively higher leverage near the tail of the loan increases the severity of defaults compared to the defaults in the earlier periods. This is because the tail is very short and the mean CFADS stays constant. Therefore, if a default occurs near the maturity of debt, there may not be enough cash in the tail to cover the losses. That is, the defaults nearer to the maturity of the debt can be more costly than those during earlier periods.

In the case of the rising DSCR family, the tail is relatively longer and CFADS is rising, hence there is a lot more cash available in the tail of the project. Therefore, the severity of losses is much less affected by the timing of defaults.

We see the effect of different tail values further in the distribution of deaths. We use 'death' to denote the outcome where the project company ceases to be a going concern upon default i.e. there is no recovery from default. This happens when cash available upon default, including the cash in the reserve accounts, exceeds the value of SPE in operation. Thus, debt holders are better off taking the available cash, and letting the SPE go bankrupt. In the case of the rising DSCR family, we do not see any deaths. This is because the extra CFADS in the tail makes SPEs more valuable as a going concern. While for the flat DSCR family, lower value in the tail makes it more likely for a hard default to lead to death near the maturity of the project, as there is not a lot of cash left in the remaining periods. Note however that the probability of death remains very low at around 0.5% only after year 22.

32 - Moody's definition of default only includes hard defaults, and the projects that default more than once are only counted once (Moody's, 2014)

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Figure 20: Exit value of lenders, and total firm and debt values

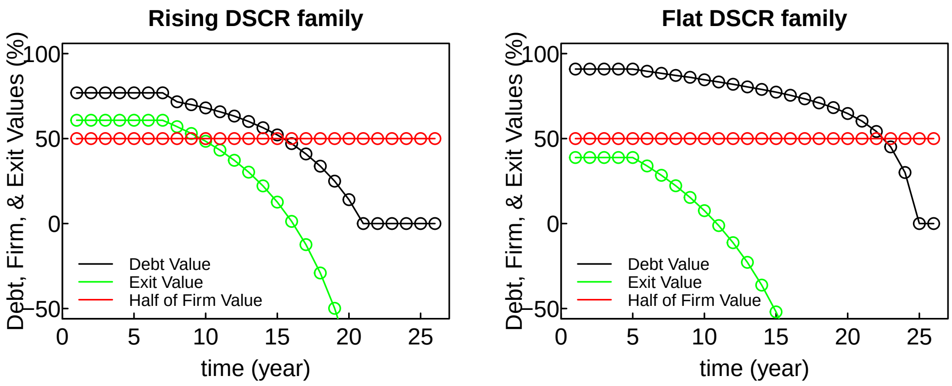
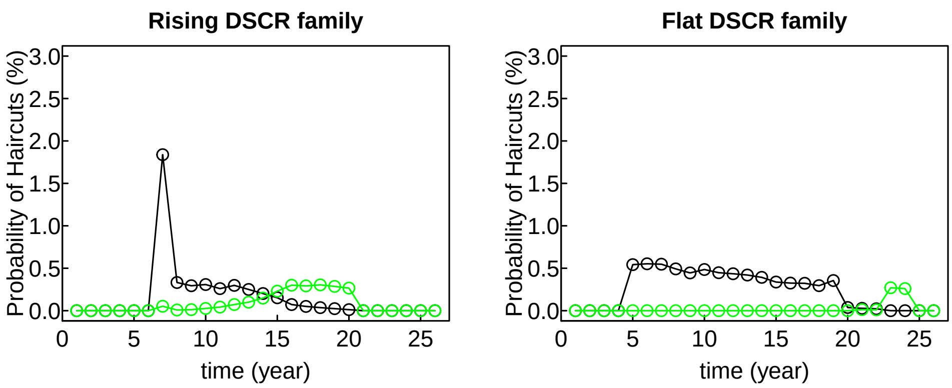


Figure 21: Probability of lender and equity haircuts (of any size)



5.3 Output of the renegotiation model

Next, we highlight the output of our debt renegotiation model in the event of a hard default, given the assumptions made in section 5.1.

Figure 20 shows the evolution of lenders' *exit value* and debt value in time. Earlier in chapter 3, we have defined the exit value as the value of the firm minus exit costs (such as finding a new equity investor or selling the project) at time *t*. We also showed that the outcome of renegotiation after a hard default is determined by lenders' exit value.

As indicated on figure 9 on page 46, lenders gradually shift from regions in which their exit value is relatively higher (because the SPE still has many years to live), to regions in which they are less and less likely to choose to exit and more likely to renegotiate and, potentially, take a haircut (but losing less than by choosing an exit).

On figure 20, the black line indicates the evolution of the total (market) value of debt at time *t* in both the rising and flat DSCR families, and the green line indicates the exit value of lenders upon a hard default (the exit value of equity holders is always zero).

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Finally, the red line indicates a threshold of 50% of the firm's value at time *t*, which we have assumed to be the minimum that lenders would get out of a renegotiation following a hard default.

In the case of the rising DSCR family, the exit value is higher than the 50% threshold but lower than the total debt value during the first part of the project's life. This corresponds to regions 4 in figure 9 (region 3 is ignored in the simulation). After some time, the exit value is less than 50% of the firm and a few years later, the value of debt is also less. In these last two cases, lenders are in region 5 on figure 9 i.e. they would rather take 50% of the firm post-restructuring and they are thus less and less likely to take a haircut (since the debt is worth less and less as a share of the firm's value).

In the case of the flat DSCR family, the exit value is always less than the 50% threshold and so lenders find themselves having to renegotiate and potentially take a haircut in the event of a hard default. However, as figure 21 illustrates, such a haircut is also very unlikely.

Figure 21 shows the (unconditional) probability for lenders or equity holders to have to take a haircut during the life of the project following a hard default. The probability of lenders having to write down some of their investment following a hard default is higher in the earlier part of the project because the outstanding debt amount represents such a large proportion of the firm's future value and their exit value is always lower than the value of the debt (which means that exit would always be costly).

Later, lenders can always get at least 50% of the firm and thus never have to agree to a lower debt service.

In the case of the rising DSCR family, the likelihood that equity investors have to let go of some of their investment upon a hard default increases in time as the lenders' decreases. Indeed, as lenders take over the project upon a hard default in its later phases, the outcome of renegotiation (50% for the lenders) becomes increasingly costly for equity owners.

We note that under the assumptions made in the simulation, lenders never enter the more extreme regions of figure 9 (regions 2 and 1). However, these could be accessible under different assumptions about the relative size of exit and renegotiation costs.

5.3.1 Recovery Analysis

Next, we show the time evolution of loss given default (LGD) and duration at time *t*, assuming that the base case debt payments are realised until $t - 1$. That is, we move forward in time, assuming that base case debt payments are realised and compute the LGD, the expected recovery rate (1-LGD) and duration at that point in time.

Figures 22 and 23 show LGD as a percentage of existing value of debt and in absolute terms, respectively. For the rising DSCR family, LGD decreases in time, as the distribution of losses does not change much during the loan's life. For the flat DSCR

5. Results

Figure 22: Loss given default (present value of expected losses) as a percentage of the value of debt.

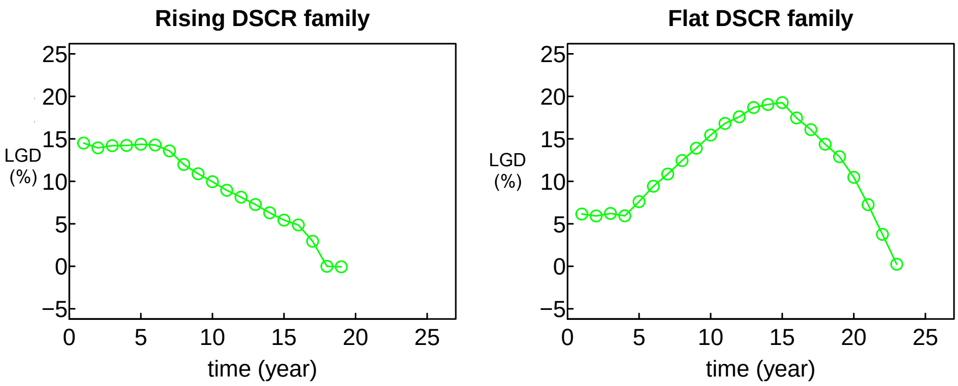


Figure 23: Loss given default (present value of expected losses) in absolute terms for a $1,000 investment.

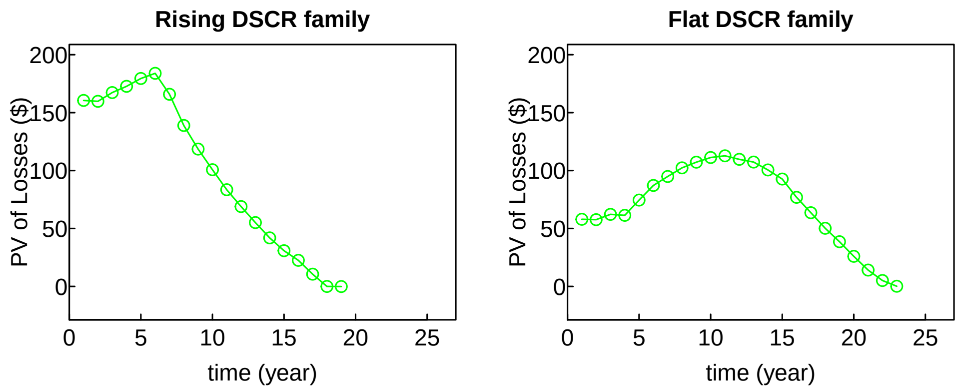
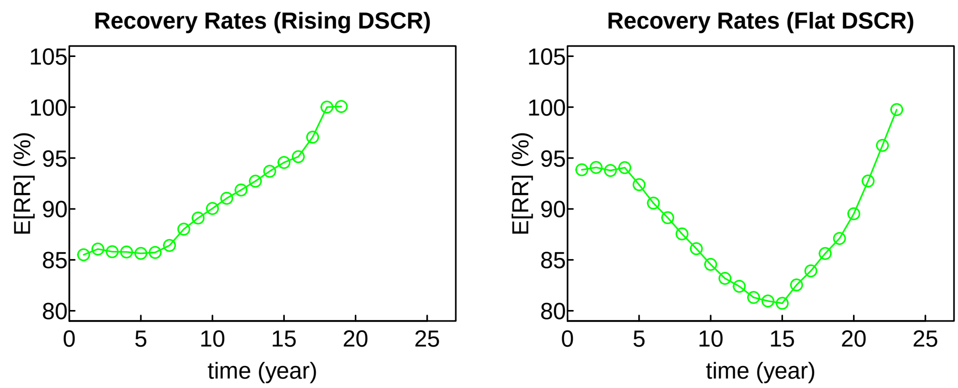


Figure 24: Recovery rates



5. Results

family, however, the LGD first increase, and then decrease.

This increase LGD for the flat DSCR family arises from the increasing severity of losses near the maturity of the loan as observed in figure 19, where mean EL, VaR, and cVaR all increase linearly towards to maturity of the loan. Hence, LGD, which is affected by the full distribution of the losses and not just by mean losses, increases in time as we approach the period of the most severe losses. As we move through time, expected losses continue to increase due to the more extreme losses getting nearer, but also decrease due to the losses that now lie in the past and were not realised. At some point, the latter effect dominates and LGD begin to decrease.

Figure 24 shows the *expected* recovery rates as a function of time. Notice, however, that the final realised recovery rate is 100% in our example as we assume that base case payments are received in every period, and no default occurs — neither technical nor hard. As expected, recovery rates are always very high (always above 85%).

5.3.2 Duration

Figure 25 shows the time evolution of effective duration (see section 7.2.5 for the definition of duration) for the two families. Both families show a largely similar trend.

Figure 26 shows the relationship between losses and duration upon a hard default, when the value of debt is given by the outcome of renegotiation, but the choice of debt schedule can be in debt holders' control. That is, debt holders can choose amongst various debt schedules that have the same value at the time of default.

In this figure, we show how losses and duration are affected by the choice of debt schedule. Each point in the figure is obtained be setting a maturity for the new debt schedule, and then computing the debt payments so that the value of debt schedule is equal to the value of debt as determined by the debt renegotiation model. Debt schedules that lead to lower losses are the ones with higher duration (and longer maturity). However, loans with very short tails embodied by the flat DSCR family do not allow trading-off lower credit risk for a longer duration, only to increase credit risk for a shorter duration.

This trade-off exists because in order to reduce expected losses, the DSCR has to be kept sufficiently high, which decreases the potential size of renegotiated debt payments, and as a consequence increases duration. [33]

Hence, there exists a trade-off in infrastructure project finance debt between credit risk and duration risk.

5.4 Return Measures

In this section, we discuss the relevant return measures for the two DSCR families: yield, and z-spread both for a benchmark investor with a Sharpe ratio of 1, and for the investors at the two extremes of our Sharpe ratio band [0, 2].

33 - This argument would not hold if liquidation costs are so low that the debt holders can benefit from default. In this case a debt schedule with lower duration would also have a lower credit risk. However, we ignore this possibility, as liquidation costs are unlikely to be sufficiently low.

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Figure 25: Time evolution of duration.

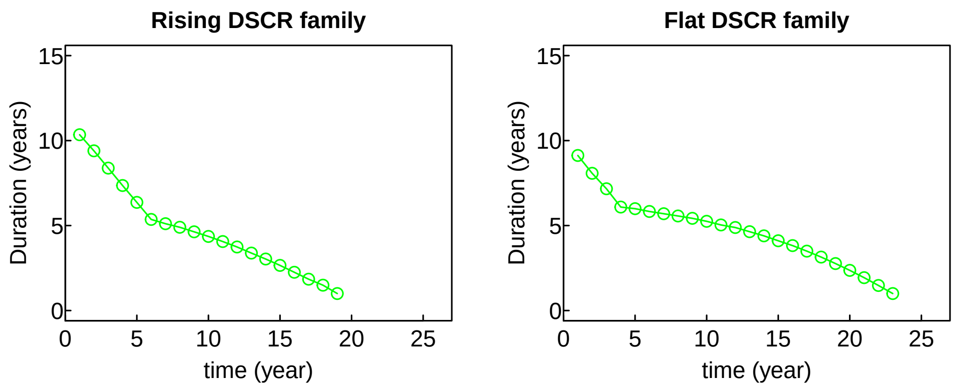

Figure 26: Traded-off between credit and interest rate risk. The x-axis shows the duration relative to the mean duration, and the y-axis shows the loss relative to the mean loss.

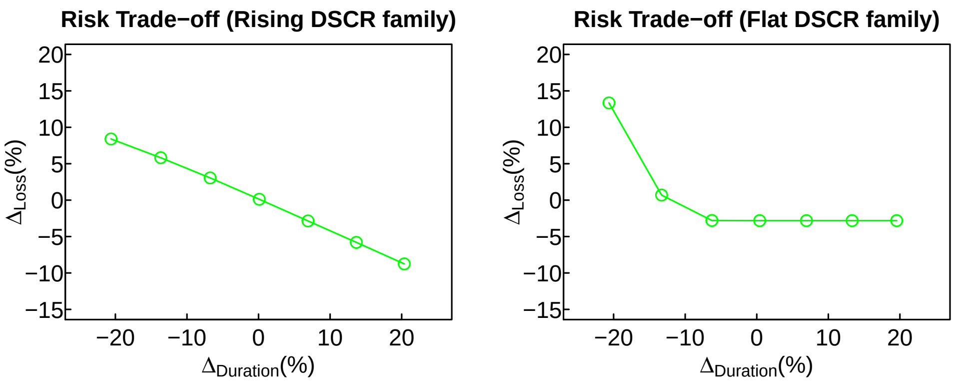

Figure 27: Comparison of yield, and z-spread for the two DSCR families.

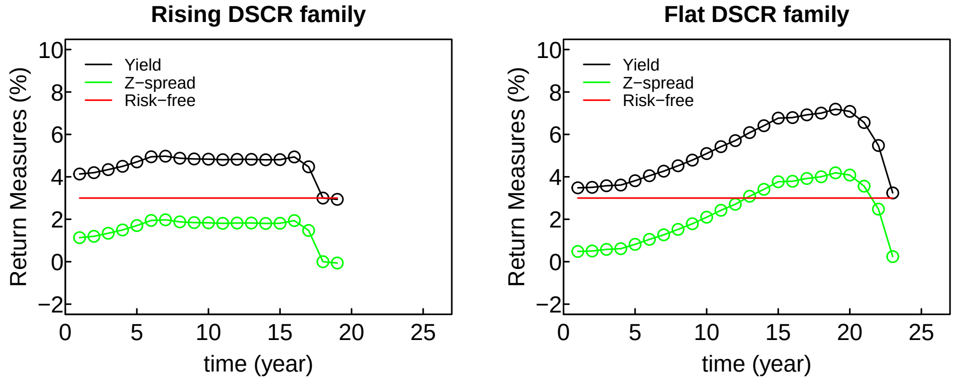

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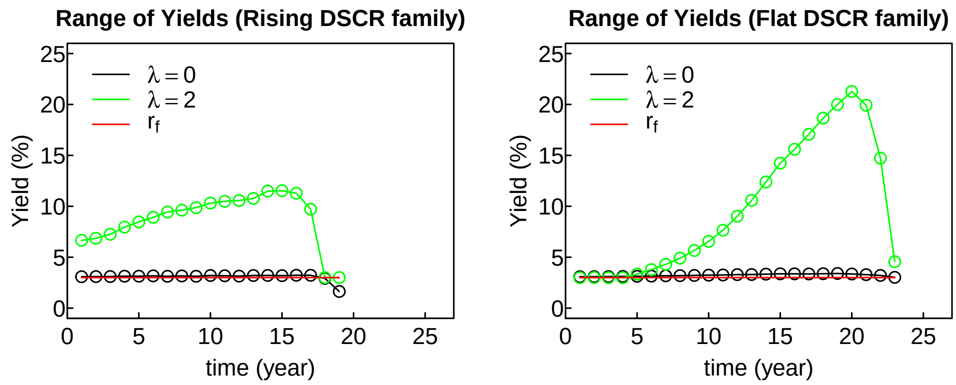
Figure 28: Range of yields for the two DSCR families.



The yield is calculated as

$$V_t^D = \sum_{i=t}^{T_D} e^{-y_t(i-t)} \mathrm{DS}_i^{\mathrm{BC}} \qquad (5.1)$$

where y_t is the yield at time t. And the z-spread is simply the difference between the yield at time t and the risk free rate at time t.

Figure 27 compares the yield and z-spread for the two families: the yield for the rising DSCR family largely stays at the same level, while the yield for the flat DSCR family increases in time before peaking and plunging towards the risk free rate near the loan's maturity.

This difference arises due to the different loss profiles. As can be seen in figure 22, the expected losses increase in early periods for the flat DSCR family, which increases its yield. That is, increasing expected losses decrease the debt value faster than the debt payments alone would have, and hence result in a higher yield. Near the end of the loan's life, when expected losses stop increasing, the yield also stops increasing, and then converges towards the risk free rate as the value of expected losses reaches zero near maturity.

For the rising DSCR family, the expected present value of losses increase until the first payment (year 5), and decrease linearly. Hence, the yield also increases in the first few years, and then stabilises at a constant level above a risk free rate. Near the maturity of the loan, when the expected losses for the rising DSCR family go to zero, the yield also approaches the risk free rate.

Finally, in figure 28 we show the range of yields for the two extreme values of the required Sharpe ratio. While for the flat DSCR family, both yield curves remain above the risk free rate, the lower bound on the yield curve for the rising DSCR family falls below the risk free rate near the maturity of debt. This difference arises due to differences in the tail values. For the flat DSCR family, the tail value is limited and there is little scope for rescheduling the debt near the maturity. This can also be seen from figure 18, where the probability of

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hard default is equal to the probability of death near the debt maturity, indicating that almost all hard defaults lead to death near the maturity of debt. Therefore the value of debt near maturity is determined simply by its scheduled debt payments, as there is no scope for rescheduling.

In the case of rising DSCR family, the tail value is sufficiently high even near maturity. As a result, debt owners can reschedule their debt upon default and get more than the promised debt payments. Therefore, the value of debt exceeds the scheduled debt payments near the debt maturity, and the yield falls below the risk free rate. This effect is more pronounced for an investor with a low level of risk aversion, as this leads to higher values of debt and a lower yield, all else being equal.

6. Conclusions

6. Conclusions

At the beginning of this paper, our objectives were to determine the most appropriate pricing model for infrastructure project finance loans and to design a methodology that can be readily applied given the current state of empirical knowledge and later at a minimum cost in terms of data collection.

Thus, we have designed the first methodology to compute relevant risk and return measures for long-term investors in illiquid infrastructure debt and prudential regulators. Our framework allows the computation of default frequencies, expected loss, expected recovery rates, loss given default, value at risk, expected shortfall, effective duration, yield, and z-spread.

We have also defined the minimum data collection requirement for infrastructure project loan valuation.

6.1 Summary of the Methodology

Our framework for the valuation of project finance loans consists of a **cash flow model** that uses observable DSCR data to model the expected free cash flow of the SPE, including a renegotiation model to model the changes in debt schedules upon reorganisations upon default, and a risk-neutral **valuation model** taking into account the incompleteness and frictions in the market for infrastructure assets.

The framework is flexible enough to incorporate the the endogenous nature of credit risk in project finance loans including the embedded options created by covenants, step-in rights, reserve accounts, or cash sweeps.

6.1.1 The Cash Flow Model

To model DSCR dynamics, we consider two broad families of generic infrastructure project financing structure based on their *ex ante* DSCR profile: a "rising DSCR" family of loans and a "flat DSCR" family. We argue that the underlying risk profile is different between the two families, and reasonably homogenous within a single family.

The rising DSCR family is mostly associated with projects that have higher revenue or market risks, and the flat DSCR family is typically associated with projects with lower revenue risks. Existing empirical research on the determinants of credit spreads in infrastructure project finance (see for example Blanc-Brude and Ismail, 2013) suggests that the projects within each family can be reasonably expected to form a homogeneous population. These ideal types then serve as the basis of our DSCR model.

In the absence of empirical observations, we use a lognormal distribution with a constant mean and volatility of the change in DSCR for the rising DSCR family, and a normal distribution with a constant mean and standard deviation of the DSCR for the flat DSCR family. We then infer the CFADS dynamics using the base case debt schedule and the DSCR model.

With a known CFADS process, cash flows for a given debt schedule can be modelled according to the priority of payments estab-

6. Conclusions

lished in the original debt contract. The debt schedules can, however, change upon reorganisations following technical or hard defaults. Thus, we model these changes in debt schedule through a debt restructuring model that takes into account the relative bargaining powers of debt and equity holders upon technical or hard defaults, and determines a new debt schedule as an outcome of the renegotiation process.

Thus, we can project cash flows to lenders in all states of the world (using a Monte Carlo simulations)

6.1.2 The Valuation Model

Next, loan cash flows are valued using a risk-neutral framework, adjusting for investors' risk preferences by risk neutralising the DSCR distribution. For risk averse investors, these risk-neutral probabilities penalise the riskiness of cash flows by decreasing the expected value of risky cash flows. That is, instead of discounting the actual expected cash flows at a premium above the risk-free rate, expected cash flows are decreased under the risk-neutral measure and discounted at the risk-free rate.

This shift in the expected value of cash flows is determined by the premium required for one unit of risk, which reflects the investor's level of risk aversion. The more risk-averse an investor, the higher the premium demanded for each unit of risk, and the lower the expected value under the risk-neutral probability measure. Hence, The task to determine the risk-neutral probability measure consists of estimating the required premium per unit of risk.

However, in the absence of market prices, as is the case with illiquid infrastructure debt, there is no unique value to which the discounted risk-adjusted cash flows should correspond. Instead, incorporating investors' preferences towards risk to determine the cumulative value of expected cash flows must lead to a range of values, since the required price of each unit of risk can depend on individual investors' unique circumstances, including regulatory requirements, the diversification level of the existing portfolio, or the structure of their liabilities.

Still, we argue that the required prices of risk would always lie in an 'approximate arbitrage band' of $[0, 2]$ that rules out investments that are either too risky for any any investor to take, or too attractive to survive arbitration. The lower limit of the band corresponds to an investor that requires no premium above the risk free rate for bearing the risks in PF loans. While the upper limit corresponds to an investor that requires a premium of 200 bass points for bearing each unit of risk (one standard deviation of the DSCR) taken in a PF loan. We argue, from both a theoretical and an applied perspective, that Sharpe ratios above this upper limit would to be too good to be true ("good deals") and thus cannot exist. Hence, we determine a pricing band for PF loans and the corresponding risk measures.

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Finally, risk adjusted cash flows to debt holders discounted using risk-free rates are combined using the Black Cox decomposition to take into account the path dependency of PF debt as determined by the renegotiation model. The Black-Cox decomposition determines the value of a security as the sum of four parts: its value at maturity, at a lower (default) and an upper (refinancing) reorganisation boundary, and its cash flows before reaching maturity or getting reorganised at any of the boundaries.

6.1.3 Data Collection Requirements

Our methodology only requires a parsimonious dataset as input. The key model inputs are given in table 3.

Most of the data point presented in the table only needs to be collected once at the financial close stage and are used to allocate observations to different generic project types such as the ones we discussed in this paper. After financial close, we need to collect DSCR data and answer a few questions about the status of the project (default, lockup, etc.) to better calibrate the probabilities of being in any one state.

This data is or can be routinely collected by lenders in project finance since it is part of the original final financial model or is the object of monitoring during the life of the loan.

With such empirical observations, the distribution of $DSCR_t$ can be calibrated to best reflect the state-dependent cash flow dynamics of infrastructure project debt.

Our approach requires the ability to partition the universe of infrastructure project finance into a limited number of tractable generic cash flow models. While individual projects can be very idiosyncratic, we argue that a limited number of reasonably homogeneous families of structures can be established. Within such groups, individual projects may not be the same but they are expected to exhibit more commonalities that with project in other families.

The case for partitioning infrastructure debt instruments by type of project revenue risk is very strong, as this has been shown to explain credit spreads very well in existing empirical research. In fact, project revenue risk profiles correspond to specific financial structuring choices made by lenders that (initial leverage, tail, and DSCR trajectory) and that, in turn, these choices signal relatively homogenous underlying cash flow volatility.

Figure 29 provides an illustration of a simple partitioning of the different types of *a priori* cash flow risk profiles into "building blocks" that can be modelled and calibrated with available cash flow data.

On figure 29, individual loans are categorised by generic category as a function of the structuring decisions of lenders, who, having done extensive due diligence before financial close about the type of risks to which the project will be exposed, *reveal this information* in their choice of financial structuring. Here, in a simple framework, project loans can

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Table 3: Data collection requirements.

Collection stage	Data points
Collected once at financial close	- Base case debt service and calendar - Base case $CFADS_t$ (optional), $DSCR_t$, ADSCR, PLCR - Covenants (reserve accounts, cash sweep, technical default triggers (minimum DSCR or LLCR), etc) - Initial senior and subordinated debt, initial equity - Foreign exchange mismatch (y/n), Interest rate swap (y/n) - Project dates, life, construction start and completion dates - Country, sector (finite list to be determined) - Revenue risk profile (merchant, contracted, mixed) - Guarantees (Grantor, ECA, PRI, etc.) - ESG (Equator Principles: y/n, A/B/C)
One-off events	- First drawdown (date) - First debt service payment (date) - Construction start (date) - Construction completion (date)
Collected in time (annual or bi-annual periods)	- DSCR value - Refinancing (y/n) - Lockup (y/n) - Technical default, inc. Basel-II definition (y/n) - Hard default (y/n) - Emergence from default (y/n) - Lender take-over of the SPE (y/n) - Loan sale (y/n) or SPE bankruptcy (y/n)

have either a high or low average DSCR (ADSCR), either a flat or rising DSCR time profile, and either a large or a small tail. This structure can then be placed in another matrix representing time (the lifecycle of the project, e.g. whether it has already been built or not) and space (the impact of country-specific risks).

By partitioning the infrastructure project finance universe into a parsimonious set of tractable cash flow models, which can be calibrated using available data, we can create the *building blocks* thanks to which the performance of different exposures to infrastructure debt can be identified and, eventually, portfolio (benchmark) construction can take place.

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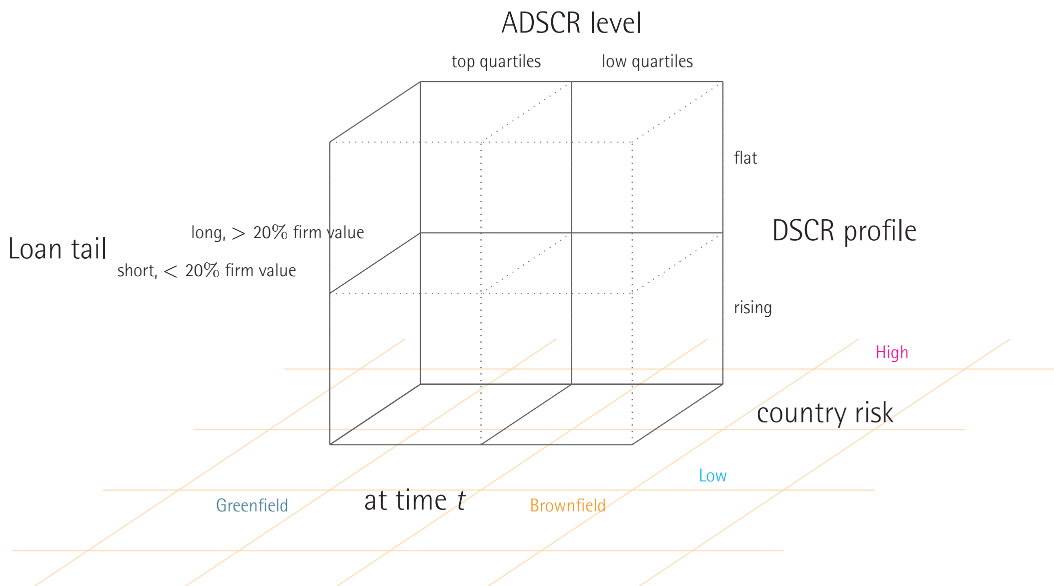
Figure 29: Example partitioning of infrastructure project finance debt cash flow dynamics into "building blocks"



6.2 Findings

Here, we report results for a typical investor requiring a Sharpe ratio of 1 to invest in illiquid infrastructure debt. Interestingly, this level of correction of the expected cash flows under the risk-neutral measure yields probabilities of default that are in line observed default frequencies values reported by rating agencies, as we detail below.

A low but dynamic risk profile

We find that the debt of both types of generic infrastructure projects discussed in this paper — merchant and contracted — exhibit highly dynamic risk profiles.

In the case of merchant infrastructure projects, the probability of both technical and hard defaults (PD), and of hard defaults only (Moody's definition), goes down sharply post construction, while mean (EL) and extreme (VaR,cVaR) expected losses tend to rise throughout the loan's life. Similarly, in the case of contracted infrastructure projects, while PD stays almost constant during the loan's life, the severity of losses increase with time.

The diverging trends in the distribution of defaults and losses are a consequence of restructurings upon defaults. Even if defaults are concentrated in a certain period of time, debt restructuring can spread losses over the entire life of the project. Hence, losses tend to increase with time, as the cumulative number of defaults (and hence restructurings) accrue losses near the end of loan's life. However, part of the losses suffered during the loan's life are , recovered in the loan's tail, thus reducing the overall expected loss.

Indeed, risk levels are found to be relatively low and recovery relatively high. While EL never rises above 2%, VaR and CVaR while

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they increase towards the end of the loan's life as the value of the tail is exhausted, never reach levels higher the 6% and 10% respectively, while expected recovery rates are always in the 80% to 100% range.

Hard default frequencies match reported averages

The different aspects of the projects' risk profile can largely be explained by their DSCR profiles, tail values, and the costs of exit relative to the cost of renegotiation for lenders.

The rising DSCR profile of merchant infrastructure implies that the project's likelihood of default decreases faster in time. If a loan survives the first few years after the construction stage, the increasing mean DSCR more than offsets the increasing DSCR volatility, making it less likely that the project will default in the future. For contracted infrastructure, flat DSCR profile implies that the probability to default barely changes in time, though it stays at a very low level due to lower DSCR volatility.

Moreover, when using Moody's definition of default in project finance – by which each loan is only allowed to default once (Moody's, 2013) – we find marginal default frequencies in line with reported empirical estimates, trend downwards from just under 2% at the beginning of the loan's life to almost zero after ten years, in the case of merchant infrastructure, and flat at 0.5% for contracted projects.

While Moody's (2013) does not explicitly differentiate between merchant and contracted projects, its main sample is effectively dominated by merchant or part merchant projects, yielding the oft-reported decreasing PD profile reproduced here on page 32; while in a separate study focusing on PPPs – effectively contracted infrastructure – Moody's report a flat PD in the range indicated here.

Low credit risk and high recovery

The loss profiles for the two DSCR families are similar insofar as expected losses (EL) are very low but increase towards the maturity of the loan, but differ in terms of the behaviour of extreme losses. Extreme losses (VaR and cVaR) increase almost linearly towards the maturity of the loan for contracted infrastructure projects, but stay relatively constant near the loan's maturity for merchant projects.

The increasing EL for both DSCR families is a consequence of cumulative haircuts received upon hard defaults in all the prior periods. The increasing VaR and cVaR in the case of flat DSCR family are due to a lower tail value, and constant leverage in time, the combination of which implies that near the loan's maturity the remaining value of the project may not be sufficient to recover losses, making defaults more severe.

The evolution of the loss given default (LGD) i.e. one minus the recovery rate, as a function of time. Recovery rates are very high (always above 85%).

For merchant infrastructure (rising DSCR), LGD decreases in time, as the distribution of losses does not change much during

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the loan's life. For contracted infrastructure however (flat DSCR), the LGD first increase, and then decrease.

This increase in LGD for the flat DSCR family arises from the increasing severity of losses near the maturity of the loan: mean EL, VaR, and cVaR all increase linearly towards to maturity of the loan. Hence, LGD, which is affected by the full distribution of the losses and not just by mean losses, increases in time as we approach the period of the most severe losses. As we move through time, expected losses continue to increase due to the more extreme losses getting nearer, but also decrease due to the potential losses that now lie in the past and were not realised. At some point, the latter effect dominates and LGD begin to decrease.

Value is driven by lenders' exit option and monitoring

Importantly, the size of losses for both DSCR families is primarily influenced by lenders' exit value net of exit costs. Exit costs determine the aggregate loss of value (debt+equity) if the debt owners take over the project company upon a hard default and do not renegotiate with the original equity investors. The higher the exit costs, the lower the value that lenders can obtain by taking over the project company after a hard default, and the lower their bargaining power in negotiations with original equity holders.

This is primarily a consequence of the unsecured nature of project finance debt, which makes the value of project company strongly dependent on the owners' ability to run it. In the absence of expertise required to run the project company, the lenders are likely to be forced to offer concessions to equity holders to benefit from their ability to run the firm. Hence, lenders may have to suffer losses even in otherwise low risk projects like contracted infrastructure because replacing the equity owners upon a hard default, while it is in their power, may be very costly.

As a consequence, ongoing monitoring of the SPE conducted is required of lenders in project in order to avoid ever having to contemplate exercising their option to exit, in particular, technical default triggers (e.g. a low DSCR or loan-life cover ratio) allow lenders to intervene and maximise their recovery rates long before more expensive options to restructure, sell or liquidate the SPE ever arise.

A DSCR-driven yield profile

The yield curve for both types of project debt is driven by two forces: the increasing severity of losses towards the end of the loan's life pushes up the yield since the discounted value of expected cash flows is further reduced, while the sequential resolution of uncertainty as maturity approaches pulls it down. The actual yield curve balances the two effects.

Initially the yield goes up as we get closer to the region where larger losses are likely to be accrued and the first effect dominates. However, as we move past this region, the probability of default during the remaining life of the loan goes down and expected recovery goes up: at one point the yield

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starts to decrease, as the second effect begins to dominate. In the case of rising DSCR projects, for which PD decreases more sharply and losses are more evenly distributed, uncertainty is resolved faster, and the yield begins to go down sooner in the project lifecycle.

A credit vs. duration risk trade-off

Finally, we also illustrate how the ability to reschedule debt upon technical and hard default creates a trades off between credit risk and duration risk. That is, to reduce the credit losses upon default, investors have to extend the maturity of their loan further in the tail, and have to bear a higher interest rate risk due to a higher duration.

6.3 Next Steps

In Blanc-Brude (2014), we highlight a roadmap towards the creation of long-term investment benchmarks in infrastructure. This roadmap begins with the requirement to define the underlying instruments related to infrastructure investment and to design a valuation framework that is adapted to their private and illiquid nature.

The roadmap also suggests that such a valuation framework should aim for parsimonious data inputs and to use this minimal requirement as a standard for data collection and investment performance reporting in infrastructure investment.

In this paper, we deliver the first three steps of the roadmap defined in Blanc-Brude (2014) with respect to infrastructure debt investment: defining the most relevant underlying financial instrument, designing a valuation framework that is adapted to its private and illiquid nature, and the determination of a standard for data collection and investment performance reporting in infrastructure investment.

Next steps include active data collection to better calibrate our model of $DSCR_t$ dynamics, before moving to the portfolio level of the analysis, towards long-term investment benchmark in infrastructure debt.

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7.1 Risk Neutral Measure

7.1.1 Distance to Default

Following the definition of default in project finance given in (2.2), Distance to Default for infrastructure project finance loans at time t can be defined as

$$DD_t = \frac{\mathrm{CFADS}_t - \mathrm{DS}_t^{\mathrm{BC}}}{\sigma_{\mathrm{CFADS}_t}\mathrm{CFADS}_t} \tag{7.1}$$

Using the definition of $DSCR_t$ in (2.1), the above expression can be written as:

$$DD_t = \frac{1}{\sigma_{\mathrm{CFADS}_t}}\left(1 - \frac{1}{DSCR_t}\right) \tag{7.2}$$

The above can be re-written as a sole function of $DSCR_t$ by expressing the volatility of $CFADS_t$ as a function of that of $DSCR_t$.

We have $CFADS_t = DSCR_t \times \mathrm{DS}_t^{\mathrm{BC}}$, and we know that $\sigma_{\mathrm{CFADS}_t}$ is expressed as a percentage change in the asset value, thus:

$$\begin{aligned}
r_{\mathrm{CFADS}_t} &= \frac{\mathrm{CFADS}_t}{\mathrm{CFADS}_{t-1}} - 1 \\
&= \frac{\mathrm{DS}_t^{\mathrm{BC}}}{\mathrm{DS}_{t-1}^{\mathrm{BC}}}\frac{\mathrm{DSCR}_t}{\mathrm{DSCR}_{t-1}} - 1 \\
\Rightarrow \sigma_{\mathrm{CFADS}_t} &= \sigma\left(\frac{\mathrm{DS}_t^{\mathrm{BC}}}{\mathrm{DS}_{t-1}^{\mathrm{BC}}}\frac{\mathrm{DSCR}_t}{\mathrm{DSCR}_{t-1}} - 1\right) \\
&= \frac{\mathrm{DS}_t^{\mathrm{BC}}}{\mathrm{DS}_{t-1}^{\mathrm{BC}}}\sigma_{\mathrm{DSCR}_t}.
\end{aligned} \tag{7.3}$$

Hence we can write the DD_t as

$$DD_t = \frac{1}{\sigma_{DSCR_t}}\frac{\mathrm{DS}_{t-1}^{\mathrm{BC}}}{\mathrm{DS}_t^{\mathrm{BC}}}\left(1 - \frac{1}{DSCR_t}\right) \tag{7.4}$$

where σ_{DSCR_t} is the standard deviation of the annual percentage change in the $DSCR$ value.

7.1.2 Mapping Between Risk Neutral and Physical Measures

In the Merton model (Merton, 1974), the mapping between risk neutral and physical probabilities of default is given by (Kealhofer, 2003)

$$\begin{aligned}
q(t,T) &= N\left(N^{-1}[p(t,T)] + \lambda_T\right) \\
&= N\left(-\mathrm{DD}_T + \lambda_T\right),
\end{aligned} \tag{7.5}$$

where $\lambda_T = \frac{\mu - r}{\sigma}\sqrt{T-t}$ is the Sharpe ratio for the corresponding time horizon, r is the risk-free rate, T is the maturity of the debt contract, μ and σ are the mean and volatility of returns on the firm's assets, and DD_T is the firm's distance to default at time t defined as $-N^{-1}[p(t,T)]$.

The corresponding risk neutral distribution for DSCR can be written as (Wang, 2002)

$$F^*(\mathrm{DSCR}_T) = N\left(N^{-1}[F(\mathrm{DSCR}_T)] + \lambda_T\right), \tag{7.6}$$

where $F(\mathrm{DSCR}_T)$ is the physical distribution of DSCR_T given the DSCR_t.

If the physical distribution $(F(x))$ is normal $(X \sim N(\mu,\sigma))$, or lognormal $(ln(X) \sim N(\mu,\sigma))$, then the risk neutral distribution $(F^*(x))$ follows the same distribution (normal or lognormal) with a shifted mean $\mu - \lambda\sigma$. Hence, the risk neutral distribution of the DSCR can be written as given in equations 4.7 and 4.6.

7.1.3 Decomposition of Risk Into Traded and Non-Traded Components

First, we write the current period's CFADS as

$$CFADS_{t-1} = CFADS_{t-1}^T + CFADS_{t-1}^N,$$

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where $CFADS^T_{t-1}$ represents the component of CFADS generated by the replicating portfolio, and $CFADS^N_{t-1}$ represents the components of the CFADS not generated by the replicating portfolio. Then, we write the mean return on CFADS as

$$\begin{aligned}
\mu &= \frac{E[CFADS_t]}{CFADS_{t-1}} - 1 \\
&= \frac{E[CFADS^T_t] + E[CFADS^N_t]}{CFADS_{t-1}} - 1 \\
&= \frac{CFADS^T_{t-1}}{CFADS_{t-1}} \frac{E[CFADS^T_t]}{CFADS^T_{t-1}} \\
&+ \frac{CFADS^N_{t-1}}{CFADS_{t-1}} \frac{E[CFADS^N_t]}{CFADS^N_{t-1}} - 1 \\
&= w^T_{t-1}(1+\mu^T) + w^N_{t-1}(1+\mu^N) - 1 \\
&= w^T_{t-1}\mu^T + w^N_{t-1}\mu^N \\
&= w^T_{t-1}(r+\lambda^T\sigma^T) + w^N_{t-1}(r+\lambda^N\sigma^N) \\
\Rightarrow \mu &= w^T_{t-1}\frac{\sigma^T}{\sigma}\lambda^T + w^N_{t-1}\frac{\sigma^N}{\sigma}\lambda^N,
\end{aligned}$$

where we have defined $w^{T(N)} = \frac{CFADS^{T(N)}_{t-1}}{CFADS_{t-1}}$, and $\lambda^{T(N)} = \frac{\mu^{T(N)}-r}{\sigma^{T(N)}}$.

7.2 Valuation Model

7.2.1 Black-Cox Decomposition

Under the Black-Cox decomposition, corporate securities are differentiated using four functions:

1. $P(T_D, \text{CFADS}_{T_D})$: final payment at the maturity of the contract. (We use T_D to refer to the maturity of the debt contract, which may be different from the maturity of the project denoted earlier by T.)
2. $\underline{P}(\tau, \text{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the lower boundary at time τ.
3. $\overline{P}(\tau, \text{CFADS}_\tau)$: the value of the corporate security if the CFADS reaches the upper boundary at time τ.
4. $p'(t, \text{CFADS}_t)$: the payments made by the debt security until the maturity or reorganisation.

The firm thus has four sources of value, and the contribution of each source is given below.

Using $\kappa(.)$ to denote the interval $(\underline{\text{CFADS}}(.), \overline{\text{CFADS}}(.))$, we can write the value, $h_1(V_t, t)$, of the first payout function as

$$\begin{aligned}
h_1(V_t, t) &= E\left[e^{-r_{T_D,t}(T_D-t)} P(T_D, \text{CFADS}_{T_D})\right] \\
&= e^{-r_{T_D,t}(T_D-t)} \int_{\kappa(T)} P(T_D, \text{CFADS}_{T_D}) dF^*,
\end{aligned} \tag{7.7}$$

where dF^* is the probability of CFADS_T falling between the two boundaries at time T_D.

The value of the fourth component is obtained by summing over all the payouts from time t to T_D

$$\begin{aligned}
h_4(V_t, t) = &\int_t^{T_D} e^{-r_{s,t}(s-t)} \times \\
&\left[\int_{\kappa(s)} p'(\text{CFADS}_s, s) dF^*(\text{CFADS}_s, s)\right] ds.
\end{aligned} \tag{7.8}$$

In order to determine the contribution of the second and the third components, one needs to determine the hitting times (times at which the CFADS hits a boundary), and the value of the debt security at the corresponding boundary. We denote the first

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time CFADS hits the lower boundary by $T_{\underline{\text{CFADS}}}$, and the first time CFADS hits the upper boundary by $T_{\overline{\text{CFADS}}}$. Further, let $F^*_{T_{\underline{\text{CFADS}}}}$ denote the risk neutral probability density function of the first passage time $T_{\underline{\text{CFADS}}}$, and $F^*_{T_{\overline{\text{CFADS}}}}$ denote the risk neutral probability density function of the first passage time $T_{\overline{\text{CFADS}}}$. We can then write

$$\begin{aligned} h_2(V_t, t) &= \int_t^T e^{-r_{T_{\underline{\text{CFADS}}},t}(T_{\underline{\text{CFADS}}}-t)} \times \\ &\qquad \underline{P}(\text{CFADS}_{T_{\underline{\text{CFADS}}}}, T_{\underline{\text{CFADS}}})\, dF^*_{T_{\underline{\text{CFADS}}}} \quad (7.9)\\ h_3(V_t, t) &= \int_t^T e^{-r_{T_{\overline{\text{CFADS}}},t}(T_{\overline{\text{CFADS}}}-t)} \\ &\qquad \overline{P}(\text{CFADS}_{T_{\overline{\text{CFADS}}}}, T_{\overline{\text{CFADS}}})\, dF^*_{T_{\overline{\text{CFADS}}}}. \quad (7.10) \end{aligned}$$

In order to evaluate $F^*_{T_{\underline{\text{CFADS}}}}(T_{\underline{\text{CFADS}}})$ and $F^*_{T_{\overline{\text{CFADS}}}}(T_{\overline{\text{CFADS}}})$, we need to know the stochastic process followed by the CFADS, and in order to evaluate $\underline{P}(T_{\underline{\text{CFADS}}})$ and $\overline{P}(T_{\overline{\text{CFADS}}})$, we would need to determine the payout functions at the reorganisation boundaries.

The total value of the security is then

$$V^S(V_t, t) = \sum_{i=1}^{i=4} h_i(V_t, t), \qquad (7.11)$$

where $h_i(V_t, t)$ is the value of the security at time t from the i^{th} payout function, and $V^S(V_t, t)$ is the total value of the security at time t, and V_t is the value of the SPV at time t.

7.2.2 Reorganisation at the Lower Boundary

We argue in section 3.2.3 that the renegotiation has to satisfy the three conditions given in (C_1) - (C_3). These conditions can be written mathematically as

$$\begin{aligned} &V^i_\tau(RN) \geq V^i_\tau(LQ), \text{ for } i = D \text{ and } E,\\ &V^i_\tau(RN) > \breve{V}^i_\tau, \text{ for } i = D \text{ or } E, \text{ and}\\ &V^D_\tau(RN) > V^E_\tau(RN), \end{aligned}$$

where D stands for debt, E for equity, RN for renegotiation, and LQ for liquidation. Thus, $V^i_\tau(RN)$ denotes the value of i^{th} stakeholder ($i \in [D, E]$) upon renegotiation, and $\breve{V}^i_\tau$ denotes the value of i^{th} stakeholder under no change in existing debt schedule.

The liquidation values of debt and equity can be written as

$$V^D_\tau(LQ) = \max\left(\hat{V}_\tau - L_\tau, \text{Cash}_\tau\right), \quad (7.12)$$

$$V^E_\tau(LQ) = 0, \quad (7.13)$$

where L_τ represents liquidation costs at time τ,.

We assume that the liquidation costs are constant in time, and renegotiation costs can be either 0 or R, and that debt and equity holders have identical risk preferences, and expectations about future cash flows.

Under this set of assumptions, we can have the following scenarios

1. $\hat{V}_\tau - L > V_\tau$: In this case, the liquidation value of the firm is greater than the existing value of the firm, and debt holders are better off by liquidating the firm. Hence, there will be no renegotiation in this case.
2. $max\left(\frac{1}{2}V_\tau, \text{Cash}_\tau\right) < \hat{V}_\tau - L < V_\tau$: In this case, liquidation value is higher than

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what debt holders could get by equally sharing the value of the existing firm with the equity owners. This scenario can be further sub-divided into the following scenarios

a) $\hat{V}_\tau - L > \breve{V}^D_\tau$: In this case, debt holders seek to benefit from default, and they will force equity holders to increase the value of debt to the liquidation value of the SPV. Hence the debt and equity values will be

$$V^D_\tau = \hat{V}_\tau - L, \tag{7.14}$$
$$V^E_\tau = V_\tau - (\hat{V}_\tau - L). \tag{7.15}$$

Renegotiation costs will not be incurred in this case, because if equity holders do try to impose renegotiation costs on debt holders, debt holders would simply liquidate the firm. Hence, equity holders would simply let the debt holders increase their debt value, and renegotiation will be costless.

b) $V_\tau - (\hat{V}_\tau - L) - R > \breve{V}^E_\tau$: In this case equity holders can benefit from default, and they would force the debt holders to offer concessions and reduce the value of the debt to the liquidation value of the SPV. Hence the values of debt and equity will be

$$V^D_\tau = \hat{V}_\tau - L, \tag{7.16}$$
$$V^E_\tau = V_\tau - (\hat{V}_\tau - L) - R. \tag{7.17}$$

In this case, the equity holders would have to incur renegotiation costs, because the debt holders would not lower their value unless equity holders force them to do so.

c) Neither one of the above two conditions holds: In that case neither party stands to benefit from default, and hence they would simply continue with the existing debt schedule

$$V^D_\tau = \breve{V}^D_\tau, \tag{7.18}$$
$$V^E_\tau = \breve{V}^E_\tau. \tag{7.19}$$

3. $\frac{1}{2}V_\tau > max\left(\hat{V}_\tau - L, \text{Cash}_\tau\right)$: In this case, the value of liquidation option is so low that the debt holders are better off by equally sharing the existing value of the firm with equity holders. Hence the values of debt and equity holders are given by

$$V^D_\tau = \frac{1}{2}V_\tau, \tag{7.20}$$
$$V^E_\tau = \frac{1}{2}V_\tau. \tag{7.21}$$

4. $\text{Cash}_\tau > max\left(\hat{V}_\tau - L, \frac{1}{2}V_\tau\right)$: In this case, the value of cash available in the current period is greater than the value of SPV as a going concern. Hence, the debt holders simply take the cash available at hand and the SPV ceases operations. Hence the debt and equity values are

$$V^D_\tau = \text{Cash}_\tau, \tag{7.22}$$
$$V^E_\tau = 0. \tag{7.23}$$

7.2.3 Reorganisation at the Upper Boundary

To model the outcome of reorganisations at the upper boundary, we make a few simplifying assumptions. Firstly, we ignore the effects of market conditions: level of interest rates, demand for PF debt etc, and assume that the refinancing does happen as soon as the CFADS hits a predetermined

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boundary. In other words, we assume that as soon as the CFADS crosses a certain threshold, the project's level of riskiness decreases sufficiently to justify a reduction in the cost of debt, irrespective of the market conditions. Secondly, we assume that upon refinancing, the amount of debt outstanding is paid in full along with any costs or penalties imposed by the debt covenants.

The value of debt at the upper reorganisation boundary is then given by

$$\bar{P}(\tau) = (1 + c)\left[\sum_{i=\tau}^{T_D} e^{-rate(i-\tau)}\mathrm{DS}_i\right], \tag{7.24}$$

where c is the refinancing costs, $rate$ is the original IRR of the loan, and DS_i^{BC} is the scheduled debt payment at time i.

7.2.4 Risk Measures

Here we outline the calculation of risk and return measures used in this paper.

Credit Risk

Expected loss

We use expected losses and recovery rates as a measure of credit risk. This can be measured as the difference between the present value of the base case debt service schedule and the present value of the projected debt payments. That is

$$E_t^*[\mathrm{Loss}] = \sum_{i=t}^{T} e^{-r(i-t)}\left(\mathrm{DS}_i^{\mathrm{BC}} - E^*[\mathrm{DS}_i]\right), \tag{7.25}$$

where $\mathrm{DS}_i^{\mathrm{BC}}$ is the base case debt payment in the i^{th} period, and $E^*[\mathrm{DS}_i]$ is the mean debt payment for the i^{th} period computed under the risk neutral probability measure. We compute expected losses under the risk neutral measure so that the present value of expected losses is influenced not only by mean losses, but also by the distribution of losses around the mean level.

The percentage expected loss can be written as

$$E_t^*[l] = \frac{E_t^*[\mathrm{Loss}]}{\sum_{i=t}^{T} e^{-r(i-t)}E^*[\mathrm{DS}_i]}, \tag{7.26}$$

and the expected recovery rate is

$$E_t^*[RR] = 1 - E_t^*[l]. \tag{7.27}$$

Interest Rate Risk

Interest rate risk measures the sensitivity of the value of debt due to changes in interest rates. This sensitivity can be captured by the effective duration of PF debt. Effective duration can be calculated as (Tuckman, 2002)

$$D_t = \frac{-1}{V^D(t)}\frac{\partial V^D(t)}{\partial y_t} \tag{7.28}$$

$$= \frac{-1}{V^D(t)}\frac{\partial}{\partial y_t}\sum_{i=t+1}^{T_D} e^{-y_t(i-t)}\mathrm{DS}_i^{\mathrm{BC}} \tag{7.29}$$

$$\Rightarrow D_t = \frac{1}{V^D(t)}\sum_{i=t+1}^{T_D}(i-t)e^{-y_t(i-t)}\mathrm{DS}_i^{\mathrm{BC}} \tag{7.30}$$

where DS_i^{BC} is the base case debt payment at time i, $V^D(t)$ is the value of the debt at time t, and y_t is the yield at time t.

Other Risks

Other common risks in fixed income investments are reinvestment risk, liquidity risk,

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and inflation risk. Here, for simplicity, we assume that the investors intend to hold PF debt until maturity. Under this assumption, liquidity risk would not matter. If we further assume that cash flows from PF debt securities match investors' liabilities, reinvestment risk would also not matter. However, inflation risk would still matter if the coupon payments are not inflation linked, which they typically are not. However, inflation risk is present even in the default-risk free treasury securities and the premium for inflation risk should be reflected in the risk-free term structure.

7.2.5 Return Measures

Once the value of debt has been obtained at time t, the yield can be calculated as the constant discount rate that makes the present value of scheduled debt payments equal to the current value of debt. That is

$$V_t^D = \sum_{i=t}^{T_D} e^{-y_t(i-t))} \mathrm{DS}_i^{\mathrm{BC}}, \qquad (7.31)$$

where y_t is the yield at time t.

One can also calculate the z-spread (a constant spread above the risk free term structure) at time t as

$$V_t^D = \sum_{i=t}^{T_D} e^{-(r_{t,i}+s_t)(i-t))} \mathrm{DS}_i^{\mathrm{BC}}, \qquad (7.32)$$

where s_t is the z-spread at time t.

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About Natixis

About Natixis

Natixis is the corporate, investment and financial services arm of Groupe BPCE, the 2nd-largest banking group in France with 21% of total bank deposits and 36 million clients spread over two networks, Banque Populaire and Caisse d'Epargne.

With around 15,000 employees (excl. financial stakes), Natixis has a number of areas of expertise which are organized in three main business lines: Wholesale Banking, Investment Solutions and Specialized Financial Services.

A global player, Natixis has its own client base of companies, financial institutions and institutional investors as well as the client base of individuals, professionals and small and medium-size businesses of Groupe BPCE's two retail banking networks.

Listed on the Paris stock exchange, it has a solid financial base with a CET1 capital under Basel 3 (1) of €12.7 billion, a Basel 3 CET1 Ratio(1) of 10.6% and quality long-term ratings (Standard & Poor's: A / Moody's: A2 / Fitch Ratings: A).

Natixis Global Infrastructure & Projects ("GIP") is a recognized player in the Infrastructure space. GIP has notably obtained the following rankings and awards:

- #1 Arranger in France for PPP, Concessions or DSP by Le Magazine des Affaires
- #1 Financial Advisor to sponsors in France by Décideurs
- #3 Infrastructure Bank of the Year in Europe by Infrastructure Investor
- #15 Global MLA for Project Finance by Thomson Reuters

More information on our Infrastructure expertise available on :

www.cib.natixis.com/infrastructure

(1) Based on CRR-CRD4 rules published on June 26, 2013, including the danish compromise - no phase-in except for DTAs on loss carry forwards -Figures as at March 31, 2014

About EDHEC-Risk Institute

About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of 90 permanent professors, engineers and support staff, as well as 48 research associates from the financial industry and affiliate professors.

The Choice of Asset Allocation and Risk Management

EDHEC-Risk structures all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds.

Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS); the City of London in the United Kingdom; Nice and Paris in France; and New York in the United States.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- **Core-Satellite and ETF Investment, in partnership with Amundi ETF**
- **Regulation and Institutional Investment, in partnership with AXA Investment Managers**
- **Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners**
- **Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS**
- **Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group**
- **Asset-Liability Management in Private Wealth Management, in partnership with Coutts & Co.**

About EDHEC-Risk Institute

- **Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank**
- **The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex**
- **Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)**
- **Optimising Bond Portfolios, in partnership with the French Central Bank (BDF Gestion)**
- **Asset Allocation Solutions, in partnership with Lyxor Asset Management**
- **Infrastructure Equity Investment Management and Benchmarking, in partnership with Meridiam and Campbell Lutyens**
- **Investment and Governance Characteristics of Infrastructure Debt Investments, in partnership with Natixis**
- **Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage**
- **Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers' Pension Plan**
- **The Case for Inflation-Linked Corporate Bonds: Issuers' and Investors' Perspectives, in partnership with Rothschild & Cie**
- **Solvency II, in partnership with Russell Investments**
- **Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking**

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and conferences.

Each year, EDHEC-Risk organises three conferences for professionals in order to present the results of its research, one in London (EDHEC-Risk Days Europe), one in Singapore (EDHEC-Risk Days Asia), and one in New York (EDHEC-Risk Days North America) attracting more than 2,500 professional delegates.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 58,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC's analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 1.5 million readers.

EDHEC-Risk Institute: Key Figures, 2011-2012

Nbr of permanent staff	90
Nbr of research associates	20
Nbr of affiliate professors	28
Overall budget	€13,000,000
External financing	€5,250,000
Nbr of conference delegates	1,860
Nbr of participants at research seminars	640
Nbr of participants at EDHEC-Risk Institute Executive Education seminars	182

About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance

The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world's best universities, such as Princeton, Wharton, Oxford, Chicago and CalTech, and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

Research for Business

The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of risk and investment management, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of investment management.

As part of its policy of transferring know-how to the industry, EDHEC-Risk Institute has also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

EDHEC-Risk Institute Publications and Position Papers (2010-2014)

EDHEC-Risk Institute Publications (2010-2014)

2014

- Blanc-Brude, F. and F. Ducoulombier. Superannuation v2.0 (July)
- Amenc, N., and F. Ducoulombier. Index Transparency – A Survey of European Investors Perceptions, Needs and Expectations (March).
- Ducoulombier, F., F. Goltz, V. Le Sourd, and A. Lodh. The EDHEC European ETF Survey 2013 (March).
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- Joenväärä, J., and R. Kosowski. An analysis of the convergence between mainstream and alternative asset management (January).
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- Amenc, N., F. Cocquemas, L. Martellini, and S. Sender. Response to the european commission white paper "An agenda for adequate, safe and sustainable pensions" (October).
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EDHEC-Risk Institute Publications (2010-2014)

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- Amenc, N., F. Goltz, and L. Tang. Adoption of green investing by institutional investors: A European survey (November).
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EDHEC-Risk Institute Position Papers (2010-2014)

2014

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2013

- Amenc, N., F. Goltz and L.. Martellini, Smart Beta 2.0 (June).

2012

- Till, H. Who sank the boat ? (June).
- Uppal, R. Financial regulation (April).
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